Detection Capability of Linear-and-Power Processor for Random Burst Signals of Unknown Location

Albert H. Nuttall
Surface Undersea Warfare Directorate

Naval Undersea Warfare Center Division
Newport, Rhode Island

PREFACE

The work described in this report was sponsored by the Independent Research (IR) Program of the Naval Undersea Warfare Center (NUWC) Division, Newport, RI, under Project No. B100077, "Near-Optimum Detection of Random Signals with Unknown Locations, Structure, Extent, and Strengths," principal investigator Albert H. Nuttall (Code 311). The IR program is funded by the Office of Naval Research; the NUWC Division Newport program manager is Stuart C. Dickinson (Code 102). This research was also sponsored by the Science and Technology Directorate of the Office of Naval Research, Ronald Tipper (ONR-322B).

The technical reviewer for this report was Brian F. Harrison (Code 2121).

Reviewed and Approved: 25 August 1997

Patricia J. Dean
Director, Surface Undersea Warfare

REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 25 August 1997	3. REPORT TYPE AND DATES COVERED Progress

4. TITLE AND SUBTITLE
Detection Capability of Linear-and-Power Processor for Random Burst Signals of Unknown Location

5. FUNDING NUMBERS
PE 0601152N

6. AUTHOR(S)
Albert H. Nuttall

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Undersea Warfare Center Division
1176 Howell Street
Newport, Rhode Island 02841-1708

8. PERFORMING ORGANIZATION REPORT NUMBER
NUWC-NPT TR 10,822

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Research
800 North Quincy Street, BCT 1
Arlington, VA 22217-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

A random signal (if present) is located somewhere in a time interval characterized by a total of N search bins, along with uniform noise. The signal is burst-like and occupies a contiguous set of M bins, but the location of the M bins occupied by the signal is unknown. Also, the average signal level S in an occupied bin is arbitrary and unknown.

The optimum (likelihood ratio) processor for this scenario is derived and simulated to determine its receiver operating characteristics. Practical approximations to this likelihood ratio processor lead to a class of suboptimum processors, called the linear-and-power (LAP) processors, that have a control parameter μ that can be varied for best signal detection capability. Simulations of various LAP processors reveal that near-optimum performance can be achieved by letting the control parameter μ tend

14. SUBJECT TERMS
Signal Detection   Random Burst Signals
Maximum Processor   Linear-And-Power Processor
Receiver Operating Characteristics

15. NUMBER OF PAGES
60

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

to infinity; the resultant processor, called the Maximum processor, compares the maximum of all possible partial contiguous linear sums of the observations with a fixed threshold. For search size N = 1024, the loss in detectability of the Maximum processor relative to the unrealizable likelihood ratio processor is less than 0.1 dB over the complete range of values of M, the signal burst size.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ABBREVIATIONS AND SYMBOLS

a	Parameter $1/(1 + S)$, equation (2)
A, B	Range of values of M, equation (16)
H_0	Hypothesis H_0, signal absent, equation (1)
H_1	Hypothesis H_1, signal present, equations (2) and (3)
LAP	Linear and power
LR	Likelihood ratio, equation (4)
m	Starting bin for signal burst, equation (2)
M	Actual number of bins occupied by signal (when present)
N	Total number of search bins, equation (1)
p_0	Probability density function under H_0, equation (1)
p_1	Probability density function under H_1, equation (3)
PDF	Probability density function
P_d	Probability of detection
P_f	Probability of false alarm
Q_M	Probability that signal duration is M, equation (16)
ROC	Receiver operating characteristic
S	Average signal power level in an occupied bin, (2)
v	Threshold, equation (5)
w	Weighting $S/(1 + S)$, equation (3)
$\mathbf{x}_n$	Output or observation of n-th bin, equation (4)
$\mathbf{X}_M(m)$	Partial linear sum on data $\{x_n\}$, equations (8) and (9)
α_M	Scale factor in likelihood ratio, equation (19)
γ_n	Weights, equation (12)
μ	Power applied to linear sum $\mathbf{X}_M(m)$, equation (9)
ν	Power applied to data $\{x_n\}$, equation (12)
bold	Random variable

DETECTION CAPABILITY OF LINEAR-AND-POWER PROCESSOR
FOR RANDOM BURST SIGNALS OF UNKNOWN LOCATION

INTRODUCTION

There is always a need for reliable detection of weak signals in noise. However, the problem is greatly confounded when knowledge of the signal characteristics or structure is minimal or nonexistent. Yet, this is frequently the situation in initial detection of a distant echo from an unknown target(s), when virtually nothing is known a priori.

An example is furnished by attempting to detect weak targets at long range by means of high-frequency underwater acoustic transmissions. In addition to the deleterious effects of the background noise and the reverberation (due to surface/bottom reflections and/or suspended sediments for example), the signal echo (if present) is greatly attenuated in strength due to the distance of propagation. Furthermore, the echo signal generally possesses a random structure due to the nature of the target, medium, and boundaries.

Classification and identification of target echoes cannot be reliably accomplished without first achieving reliable detection. For weak echoes especially, it is mandatory to know that the signal processing techniques being employed are operating near the absolute limits possible in the particular environments

encountered. For random signals with little structure or location information, these limits were not known and have just begun to be established (see references 1 through 6). This report continues this investigation for burst-like signals.

It is therefore necessary to formulate and solve the problem of optimum or near-optimum detection of random signals with little structure. In particular, for signals that have bursts of energy in time, but of unknown locations and/or durations as well as strength, the likelihood ratio processor, which maximizes the detection probability P_d for a specified (tolerable) false alarm probability P_f, must be derived. Alternatively, for signals that occupy globular regions in time-frequency space of unknown shape, size, and/or location, the likelihood ratio and the attendant signal processing indicated by the likelihood ratio test statistic must be derived.

Since the optimum likelihood ratio processor frequently cannot be realized practically — because it requires knowledge of unknown signal parameters, one must be satisfied with a suboptimum processor, hopefully derived as a physically reasonable approximation to the optimum processor. However, acquiescing to a suboptimum processor necessarily degrades the performance that can be attained. Therefore, it is imperative to evaluate the extent of degradation that accompanies the particular suboptimum processor adopted and ascertain its acceptability.

2

Several major technical problems must be addressed and solved. First is the actual derivation of the likelihood ratio processor under conditions that are realistic, including specifically the burst-like structure of the echo signal strength in time and/or frequency. The next task will require manipulating the exact likelihood ratio test into an approximate form or forms that eliminate any dependence on signal parameters that will not be known in practical applications. For example, the signal strength might be presumed known during the derivation of the optimum likelihood ratio processor, only to be eliminated in determining a practical suboptimum processor.

The exact level of performance attainable by the suboptimum processor is of paramount importance and must be evaluated. However, since it is suboptimum, there will definitely be a loss of performance relative to the likelihood ratio processor, which loss must be quantified. If this loss in performance cannot be ascertained exactly by analysis, it can be determined by using a combination of simulation and bounding procedures. Direct comparisons of the receiver operating characteristics (ROCs) are the best means by which useful quantitative measures of loss can be furnished.

The particular detection problem that will be investigated here is that of a contiguous burst signal of known duration or extent, but of unknown strength and location, within a specified search region of given size.

OPTIMUM PROCESSOR FOR UNKNOWN SIGNAL LOCATION

The problem will be couched in the time domain but could be extended easily to frequency or combined time-frequency space. A burst of energy is emitted and reflected off a distant distributed target of interest. The received weak signal echo (if present) is accompanied by wideband noise that occupies the entire time scale. Since the target distance is unknown, the received signal time location is unknown.

It is presumed that the received noise has been normalized to unit level (through past observations) and that a search region of N time bins has been established. The width of each time bin is related to the inverse bandwidth of the received signal burst, while the search size N depends on the uncertainty of the target range. It is presumed that the received waveform is passed through a passband filter centered on the transmitted carrier frequency, the output of which is squared-envelope detected and sampled sequentially in time. For an unknown Doppler shift, it will be necessary to broaden the filter passband.

DERIVATION OF LIKELIHOOD RATIO

The joint probability density function (PDF) of the N noise-only bin outputs, hypothesis H_0, is

$$p_0(u_1,\ldots,u_N) = \exp(-u_1) \cdots \exp(-u_N) \quad \text{for each } u_n > 0 . \quad (1)$$

For signal present, hypothesis H_1, it will be assumed that the signal duration, that is, the number of occupied bins, consists of M contiguous time slots, where M is <u>known</u>. This corresponds to a continuously distributed target in range with random properties. For equal average signal powers S in each of the M occupied bins, the conditional joint PDF of the N observations, when the initial occupied signal bin is number m, is

$$c_m(u_1,\ldots,u_N) = p_0(u_1,\ldots,u_N) \prod_{n=m}^{M-1+m} \left[\frac{a \exp(-a u_n)}{\exp(-u_n)} \right] , \qquad (2)$$

where $a \equiv 1/(1 + S)$. Therefore, the joint PDF of the observation under H_1 is, for equally likely initial starting points $m \; \varepsilon \; \{1, N+1-M\}$,

$$p_1(u_1,\ldots,u_N) = p_0(u_1,\ldots,u_N) \sum_{m=1}^{N+1-M} \frac{1}{N+1-M} \prod_{n=m}^{M-1+m} \{a \exp(w u_n)\} , \qquad (3)$$

where weighting $w \equiv S/(1 + S)$. Therefore, the likelihood ratio for random observation or measurement $x_1,\ldots,x_N$ is

$$LR \equiv \frac{p_1(x_1,\ldots,x_N)}{p_0(x_1,\ldots,x_N)} = \frac{a^M}{N+1-M} \sum_{m=1}^{N+1-M} \exp\left(w \sum_{n=m}^{M-1+m} x_n \right) . \qquad (4)$$

The corresponding likelihood ratio test is to compare the m-sum above with a fixed threshold, namely, in expanded explicit form,

$$\exp\left[w\left(x_1 + \cdots + x_M\right) \right] + \cdots + \exp\left[w\left(x_{N+1-M} + \cdots + x_N\right) \right] \overset{>}{\underset{<}{}} v. \qquad (5)$$

This same result for the likelihood ratio test was derived previously in reference 4, pages 19-21. However, no numerical results on the performance of processors (4) or (5) were presented at that time. A connection to the generalized likelihood ratio test was also made in reference 4, page 22.

Several comments about optimum processor (5) should be made. First, it is easily realized and computed (when duration M and weight w are known) because it requires only the evaluation of N+1-M exponentials, a very reasonable burden; this is in direct contrast to the case of arbitrary structureless signals, where (N-M)! operations can be required. However, since weight $w = S/(1 + S)$ depends on the generally unknown signal level S, processor (5) is not a practical processor. In addition, signal extent M is presumed known, and that knowledge is used in optimum processor form (5). Notice, also, how the known contiguous signal structure is taken into account by performing <u>linear</u> sums of the available data $\{x_n\}$, using the known duration M, before resorting to nonlinear combinations over all the equally likely possibilities.

The exponential function grows rapidly with increasing argument. This means that the largest linear sum in equation (5) will be significantly accentuated and will tend to dominate the other exponentials. This observation will lead to a suboptimum processor with potential for good signal detectability.

SPECIAL CASES OF OPTIMUM PROCESSOR

When M = 1 in equation (5), the likelihood ratio test becomes

$$\sum_{n=1}^{N} \exp(wx_n) \underset{<}{>} v . \tag{6}$$

This processor has already been investigated in reference 4, figures 2 and 12. Also, the signal has no burst duration in this case, since only one signal bin is occupied; that is, there is no useful burst information when M = 1.

When M = N in equation (5), the likelihood ratio test becomes

$$\sum_{n=1}^{N} x_n \underset{<}{>} v , \tag{7}$$

where the single exponential and the scaling w have been eliminated, because they constitute a monotonic transformation, and do not change the ROCs. Processor (7) has been analyzed in reference 1, pages 21-22 and 81-90. Also, the signal location has no uncertainty since it occupies the entire search region; that is, there is no useful burst information when M = N.

Due to the restricted behavior of these two special cases, M = 1 and M = N, and since they have already been considered previously, attention is confined here to signals with a nontrivial burst structure with some uncertainty of location, namely, 1 < M < N.

APPROXIMATE LIKELIHOOD RATIO PROCESSORS

If one denotes the inner sum on n in equation (4) as random variable $X_M(m)$, the likelihood ratio test in equations (4) and (5) can be expanded as

$$\sum_{m=1}^{N+1-M} \exp\left(w\ X_M(m)\right) = \sum_{m=1}^{N+1-M} \sum_{\mu=0}^{\infty} \frac{w^\mu}{\mu!}\ X_M^\mu(m) = \sum_{\mu=0}^{\infty} \frac{w^\mu}{\mu!} \sum_{m=1}^{N+1-M} X_N^\mu(m) \underset{<}{>} v\ .$$

(8)

Thus, the fundamental data processing on observation $\{x_n\}$ has the following typical component:

$$\sum_{m=1}^{N+1-M} X_M^\mu(m) = \sum_{m=1}^{N+1-M} \left(\sum_{n=m}^{M-1+m} x_n\right)^\mu\ ;$$

(9)

due to its form, processor (9) is called the LAP (linear and power) processor. The attractive feature of form (9) is its independence of the generally unknown weight w. A reasonable approximation to the likelihood ratio test is furnished by comparison of the statistic in equation (9) with a threshold; the choice of control parameter μ is not obvious at this point, although μ = 1, 2, 3, 4 are good candidates and should be investigated. In fact, μ need not be limited to integers in LAP processor (9). This processor was previously suggested in reference 4, page 21.

A limiting case of processor (9) is afforded by taking the $1/\mu$ root of the left-hand side, and letting $\mu \to \infty$. (The root operation is a monotonic transformation and does not change the ROCs.) The end result is the maximum processor,

$$\lim_{\mu \to \infty} \left(\sum_{m=1}^{N+1-M} X_M^\mu(m) \right)^{1/\mu} = \max_m \{X_M(m)\} \quad \text{for} \quad m \; \varepsilon \; \{1, N+1-M\} \; . \quad (10)$$

This maximum processor can also be seen to be an approximation to the optimum processor (4) or (5), since the maximum $X_M(m)$ term is exponentiated and tends to dominate the sum on m.

For $\mu = 1$, equation (9) can be simplified to the form

$$x_1 + 2 \; x_2 + \cdots + M \; x_M + \cdots + M \; x_{N+1-M} + \cdots + 2 \; x_{N-1} + x_N \underset{<}{\overset{>}{\;}} v \quad (11)$$

(for $M \leq N/2$), which indicates that the interior samples of the observation $\{x_n\}$ should be weighted more heavily. A similar behavior holds for the quadratic and cubic terms in equation (9). This suggests the following possibility as an alternative approximation to the likelihood ratio test:

$$\sum_{n=1}^{N} \gamma_n \; x_n^\nu \underset{<}{\overset{>}{\;}} v \; . \quad (12)$$

The weights $\{\gamma_n\}$ should be symmetric and peak at $n = (N + 1)/2$; their exact dependence is not obvious, but the trapezoidal weighting in equation (11) is an initial candidate. The choice of power ν could be taken in the range (1,3) at first, but need not be integer. If sequence $\{\gamma_n\}$ is chosen to be independent of M, equation (12) is a practical processor that could be applied in signal detection situations where neither the signal duration nor location are exactly known.

10

SPECIAL CASES OF LAP PROCESSOR (9)

When $M = 1$ in LAP processor (9), the approximate test takes the form

$$\sum_{m=1}^{N} x_m^{\mu} \underset{<}{\overset{>}{}} v . \tag{13}$$

This test is recognized as the standard power-law processor, which has been thoroughly analyzed in references 3-5, when power-law v there is identified with power μ here. Therefore, there is no need to investigate the $M = 1$ case here for different values of power μ in LAP processor (9).

For $M = N$ in LAP processor (9), the approximate test takes the form

$$\left(\sum_{n=1}^{N} x_n\right)^{\mu} \underset{<}{\overset{>}{}} v_1 \qquad \text{or} \qquad \sum_{n=1}^{N} x_n \underset{<}{\overset{>}{}} v . \tag{14}$$

(The removal of power μ does not alter the ROCs.) Test (14) is recognized as the standard energy detector, which has been analyzed in references 3-5; see the $v = 1$ case. Thus, there is no need to investigate the $M = N$ case here for different values of power μ in LAP processor (9). Furthermore, the ROCs for the $M = N$ test in equation (14) are independent of μ; therefore, all the burst-signal LAP processors in equation (9) have identical performances when $M = N$.

PERFORMANCE OF APPROXIMATIONS AND OPTIMUM PROCESSORS

Processor forms (11) and (12) can be analyzed exactly for any set of weights. Thus, ROCs can be constructed fairly easily for these two techniques. However, suboptimum LAP processor (9) will require simulation, due to the nonlinear (power-law) interactions that create statistically dependent random variables prior to the sum on m.

Optimum processor (5) will definitely require simulation to determine the ROCs. However, it can be simulated readily within reasonable execution time, since it requires only N+1-M exponentials per trial. This absolute upper bound on performance requires and uses knowledge of M and $w = S/(1 + S)$. It can be used to quantitatively ascertain the losses that must be tolerated when one employs the various approximate processors listed above.

The availability of an upper bound on detectability performance allows one to investigate numerous weighting schemes $\{\gamma_n\}$ in power-law processor (12), in a search for the best possible practical processor; this includes a search over power-law values ν. If the losses between the optimum and practical processors can be reduced to an acceptably low level, by choices of ν and $\{\gamma_n\}$, the latter processors can be employed with confidence, even when there is a lack of information regarding signal parameters, such as location, duration, and strength.

12

OPTIMUM PROCESSOR FOR UNKNOWN SIGNAL LOCATION AND DURATION

In this section, in addition to unknown signal location, the duration M of the signal energy burst is unknown but assumed to lie in the range $\{A,B\}$. In particular, M can take on the values $A, A+1,\ldots, B$ with probabilities $Q_A, Q_{A+1},\ldots, Q_B$, respectively. Furthermore, for a given duration M, the initial occupied signal bin m can start equally likely at one of the positions $m = 1, 2,\ldots, N+1-M$.

Under hypothesis H_0 and unit noise level, the joint PDF of the observation is again given by equation (1). Under hypothesis H_1, if M signal bins are occupied, the average signal level per bin is the same for all the occupied bins and is denoted by S_M. This dependence of the average signal level per bin on M allows for consideration of more general cases. For example, the total received average signal level is $M S_M$ if M bins are occupied; this product could be kept constant as M varies, meaning that $S_M = k/M$ in this particular case.

The joint PDF of the observation under H_1, conditioned on M signal bins being occupied, is a slight generalization of equation (3) to

$$p_1(u_1,\ldots,u_N|M) = p_0(u_1,\ldots,u_N) \sum_{m=1}^{N+1-M} \frac{1}{N+1-M} \prod_{n=m}^{M-1+m} \{a_M \exp(w_M u_n)\} ,$$

(15)

where $a_M \equiv 1/(1 + S_M)$, $w_M \equiv S_M/(1 + S_M)$. Then, the unconditional

PDF is given by

$$p_1(u_1, \ldots, u_N) = \sum_{M=A}^{B} Q_M \, p_1(u_1, \ldots, u_N | M)$$

$$= p_0(u_1, \ldots, u_N) \sum_{M=A}^{B} \frac{Q_M \, a_M^M}{N+1-M} \sum_{m=1}^{N+1-M} \exp\left(w_M \sum_{n=m}^{M-1+m} u_n\right) . \qquad (16)$$

For given data observation $\{x_n\}$ for $1 \leq n \leq N$, this leads to the likelihood ratio in the form

$$\text{LR} \equiv \frac{p_1(x_1, \ldots, x_N)}{p_0(x_1, \ldots, x_N)} = \sum_{M=A}^{B} \frac{Q_M \, a_M^M}{N+1-M} \sum_{m=1}^{N+1-M} \exp\left(w_M \, X_M(m)\right) , \qquad (17)$$

where the linear sum

$$X_M(m) = \sum_{n=m}^{M-1+m} x_n \quad \text{for } 1 \leq m \leq N+1-M , \quad A \leq M \leq B . \qquad (18)$$

Some observations on optimum processor (17) are in order at this point. This processor is relatively easily realized and computed (when signal powers $\{S_M\}$ and a priori probabilities $\{Q_M\}$ are known). However, since these quantities are generally unknown, equation (17) is not a practical processor. Notice that the known contiguous signal structure is taken into account, by performing linear sums (18) of the available data $\{x_n\}$ using the hypothesized duration M, before resorting to weighted, nonlinear combinations in equation (17).

14

To convert form (17) into a practically useful processor, one approximates the exponential according to $\exp(y) \simeq k \, y^{\mu}$, where scale factor k and power μ can be chosen to give a reasonable fit over an extended range of y. Then, equation (17) becomes

$$LR \cong \sum_{M=A}^{B} \alpha_M \sum_{m=1}^{N+1-M} X_M^{\mu}(m) \; , \tag{19}$$

where

$$\alpha_M = k \; \frac{Q_M \, a_M^M \, w_M^{\mu}}{N+1-M} = k \; \frac{Q_M}{N+1-M} \; \frac{S_M^{\mu}}{(1 + S_M)^{M+\mu}} \quad \text{for } A \leq M \leq B . \tag{20}$$

Implementation of processor (19) requires knowledge of the signal powers $\{S_M\}$ and the prior probabilities $\{Q_M\}$. If these quantities are known approximately or estimated, the scale factors $\{\alpha_M\}$ in equation (20) can be calculated. On the other hand, if some of this information is absent, it may be necessary to guess at their behaviors and use a rough estimate for the weights $\{\alpha_M\}$. If no information is available, the weights $\{\alpha_M\}$ might be taken as flat over the range $\{A,B\}$. In the latter case, the approximate likelihood ratio processor (19) becomes

$$\sum_{M=A}^{B} \sum_{m=1}^{N+1-M} X_M^{\mu}(m) \; \underset{<}{\overset{>}{\gtrless}} \; v . \tag{21}$$

The best power μ to use in tests (19) or (21) is yet to be decided on; it could well depend on M and range $\{A,B\}$, as well as the average signal level, and need not be integer.

Test (21) says to perform linear sums of the available data $\{x_n\}$ over the hypothesized duration of length M, raise it to a power, sum over all possible initial locations m, and then sum over all possible durations M within the expected range. This is a combination of a linear processor and a power-law processor. How well it performs relative to optimum processor (17) can only be determined through extensive simulation. This situation and processor has not been investigated quantitatively here; that issue is left for future study.

DETECTION PERFORMANCE OF PROCESSORS

The observed data $\{x_n\}$, $1 \leq n \leq N$, occupy a search region of size N bins, while the signal (if present) occupies M contiguous bins of unknown location within the search region. Three processors will be investigated quantitatively in this section. All require preliminary evaluation of the N+1-M linear sums

$$X_M(m) = \sum_{n=m}^{M-1+m} x_n \quad \text{for } 1 \leq m \leq N+1-M .$$

The three processors of interest are

$$\sum_{m=1}^{N+1-M} \exp\left(\frac{S}{1 + S} X_M(m)\right) \gtrless v \qquad \text{Optimum} \qquad (5)$$

$$\max_m \{X_M(m)\} \gtrless v \qquad \text{Maximum} \qquad (10)$$

$$\sum_{m=1}^{N+1-M} \left(X_M(m)\right)^\mu \gtrless v \qquad \text{Linear and Power .} \qquad (9)$$

The power μ in the LAP processor is a control parameter that can be varied for maximum detectability, while S in the optimum processor is the average signal power per occupied bin. Because the average noise power per bin is 1, S is also the signal-to-noise power ratio per bin.

The ROCs, namely, detection probability P_d versus false alarm probability P_f, for Maximum processor (10) are presented (solid curves) in figures 1 through 9 for M = 2, 4, 8, 16, 32, 64, 128, 256, 512, respectively. The label $\underline{S}$(dB) on the curves is equal to 10 log(S). These simulation results were all based on using one million independent trials of processor (10).

Superposed on each figure (dashed line) is the performance of Optimum processor (5) for one representative signal-to-noise ratio; in particular, the $\underline{S}$(dB) values investigated are 9, 7, 4.25, 2, 0, -2, -4, -6, -8, respectively. These values were chosen so that the corresponding ROC would pass close to the standard operating point P_f = 0.001, P_d = 0.5. It is immediately obvious from figures 1 through 9 that the corresponding ROCs for the Maximum and Optimum processors are virtual overlays in every case, there being a very slight gain (<0.1 dB) in favor of the Optimum processor. That is, the Maximum processor for detection of a signal of unknown strength and location is virtually optimum in terms of its performance.

It was found that the ROCs for LAP processor (9) were uniformly poorer for finite values of power μ than for $\mu = \infty$, the Maximum processor. Accordingly, the ROCs for μ = 1, 2, 3, 4 have been relegated to the appendix; results for M = 2, 4, 8, 16, 32, 64, 128, 256, 512 are presented there.

18

From figures 1 through 9, it is possible to determine what
values of signal-to-noise ratio $\underline{S}$(dB) are required to exactly
achieve the standard operating point, when using the Maximum
processor. When these values are scaled by M, the number of
occupied bins, they represent the total required received
signal-to-noise ratio. These total signal-to-noise ratios are
plotted versus M in figure 10 as the bottom curve, labeled $\mu = \infty$.
Since this Maximum processor is virtually optimum, according to
figures 1 through 9, the bottom curve in figure 10 is essentially
an absolute lower bound on all possible detection procedures
functioning under the prescribed conditions.

The remaining four solid curves in figure 10 correspond to
the LAP processor and are labeled $\mu = 1, 2, 3, 4$. These values
of required signal-to-noise ratio are extracted from the ROCs in
the appendix. They show a steady degradation in performance as
power μ is decreased from $\mu = \infty$; in fact, the $\mu = 1$ processor,
which merely sums up energies in a linear fashion, suffers a very
significant loss and should not be used, except when M is near N.
The results in figure 10 are somewhat similar to those for the
power-law processor in references 3-5, where the signal had no
structure whatsoever.

For comparison, superposed in figure 10 (dashed line, labeled
OPT-B) is the required signal-to-noise ratio for the optimum
processor for detection of a signal with no structure (see

reference 4). The difference in performance between the OPT-B curve and the $\mu = \infty$ curve represents optimal usage of the extra information available in knowledge of the signal structure, namely, a contiguous burst in this case. For M = 1 and M = N, there is no difference between these two optimum curves, because there is no extra burst information in these cases. However, for the intermediate case of M = 64, the additional burst information leads to a maximum improvement of 2.88 dB; also, as can be expected, the optimal processor form is significantly changed, from a 2.5 power-law device (in the case of no signal structure) to a Maximum operation on partial linear sums (for burst-like signal structure).
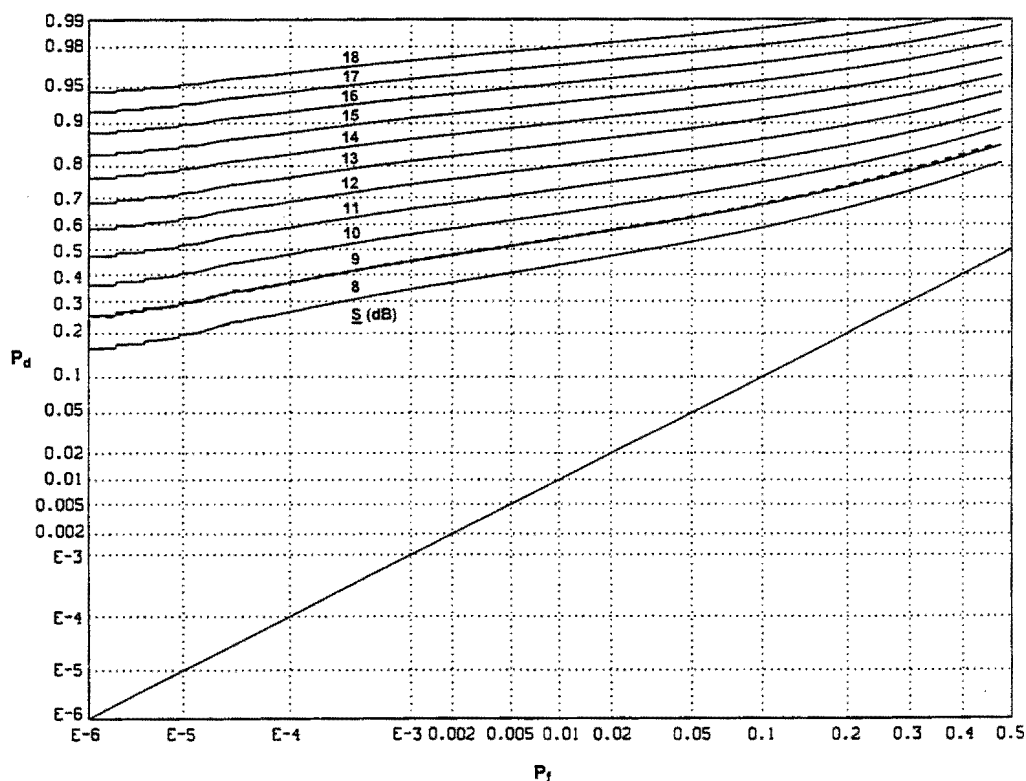
Figure 1. ROC for Maximum Processor with M = 2, N = 1024
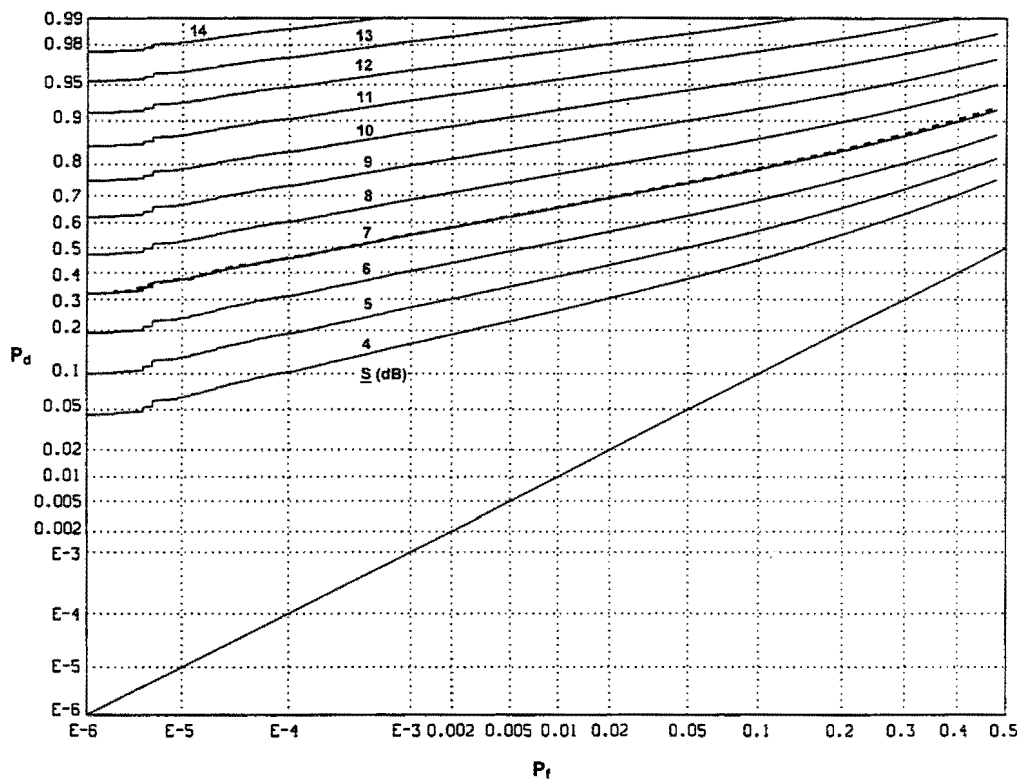
Figure 2.   ROC for Maximum Processor with M = 4, N = 1024
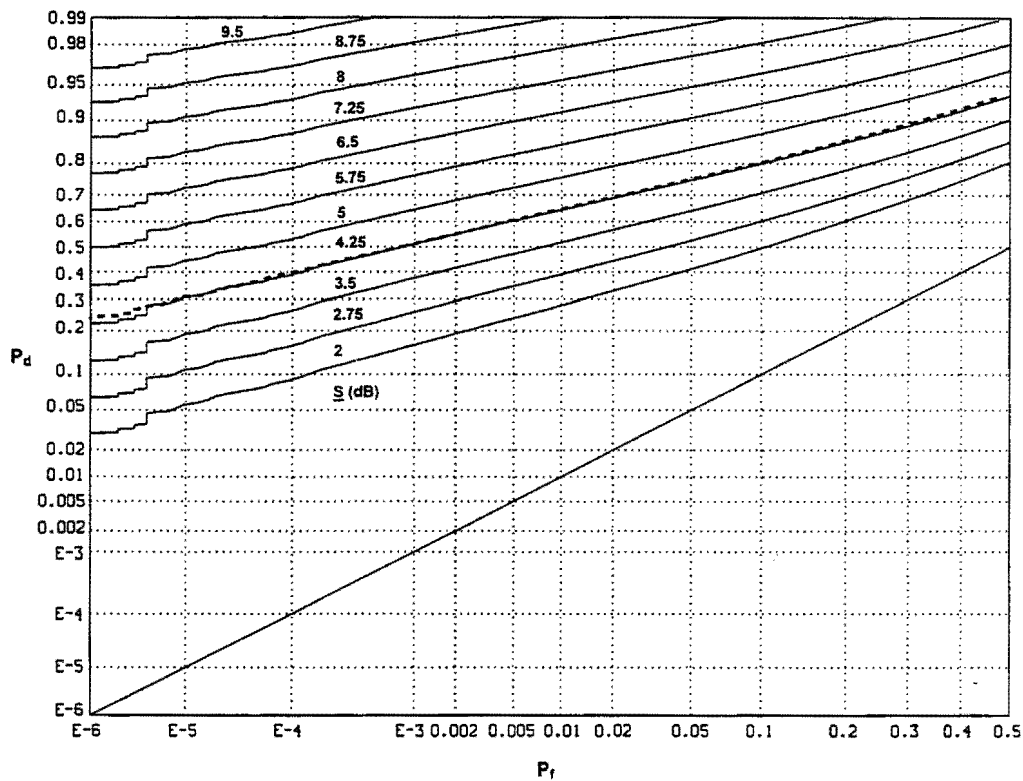
Figure 3.   ROC for Maximum Processor with M = 8, N = 1024
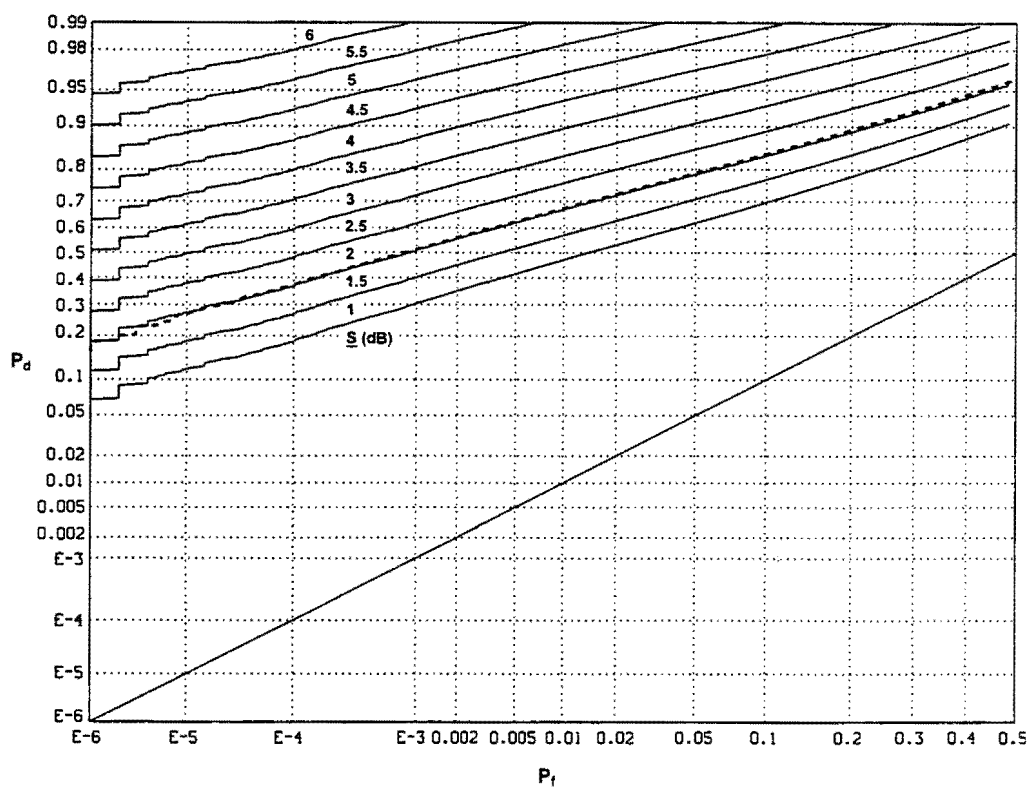
Figure 4. ROC for Maximum Processor with M = 16, N = 1024
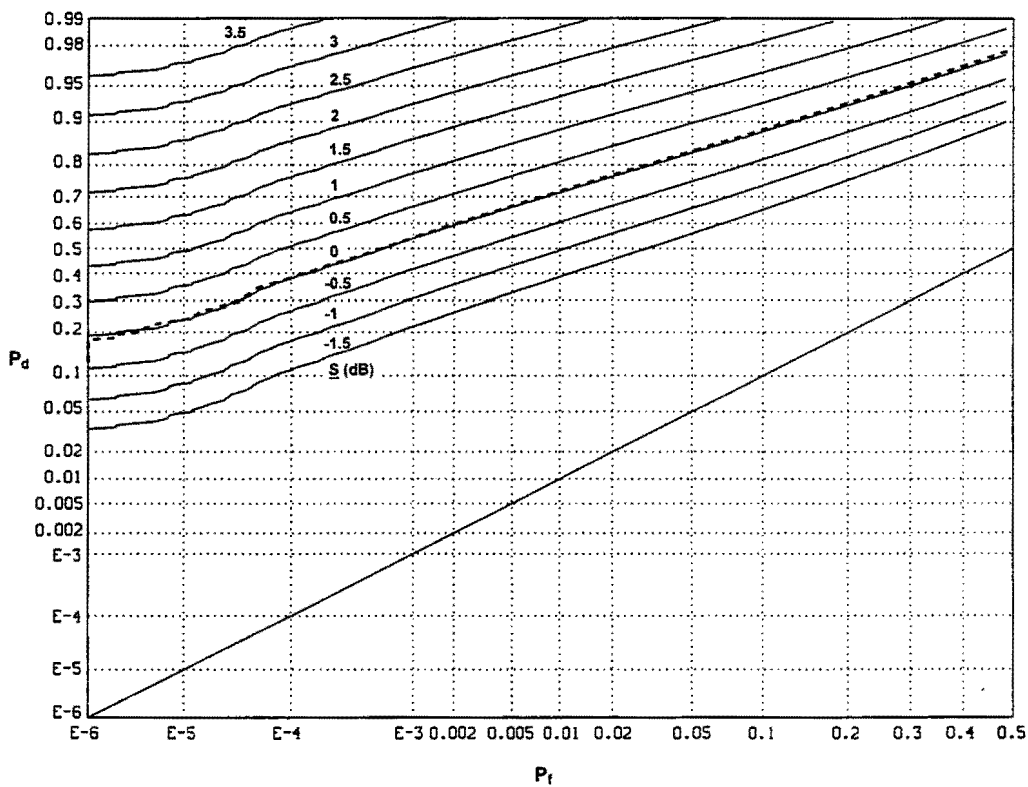
Figure 5. ROC for Maximum Processor with M = 32, N = 1024
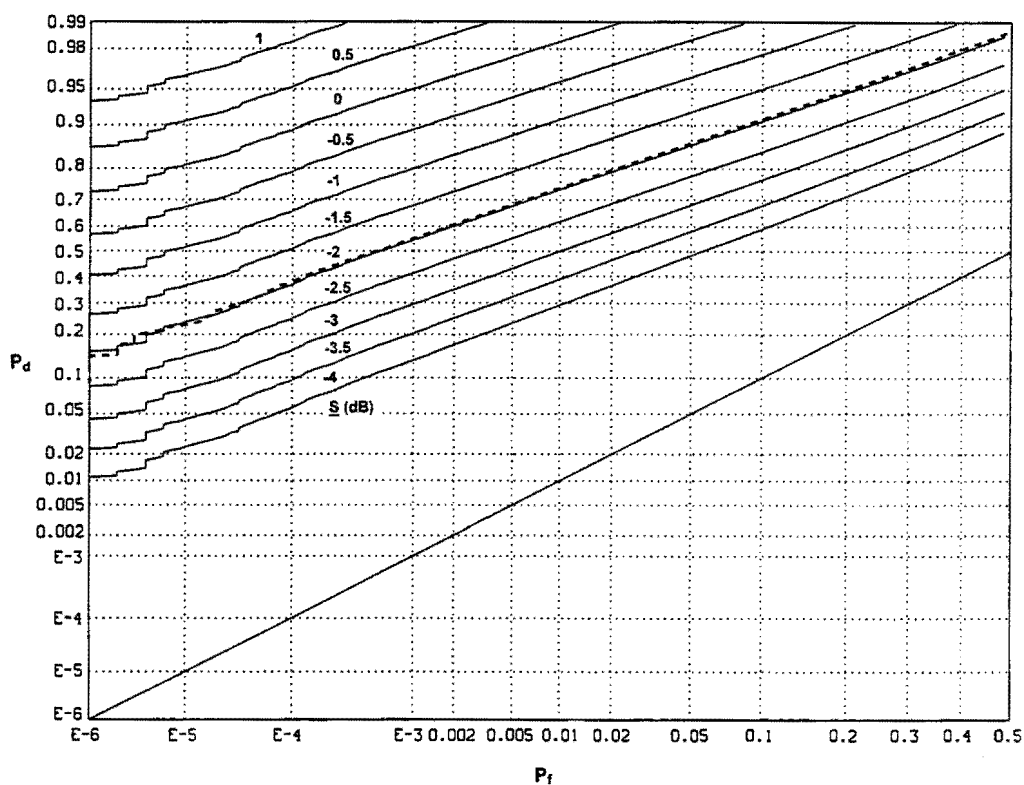
Figure 6. ROC for Maximum Processor with M = 64, N = 1024
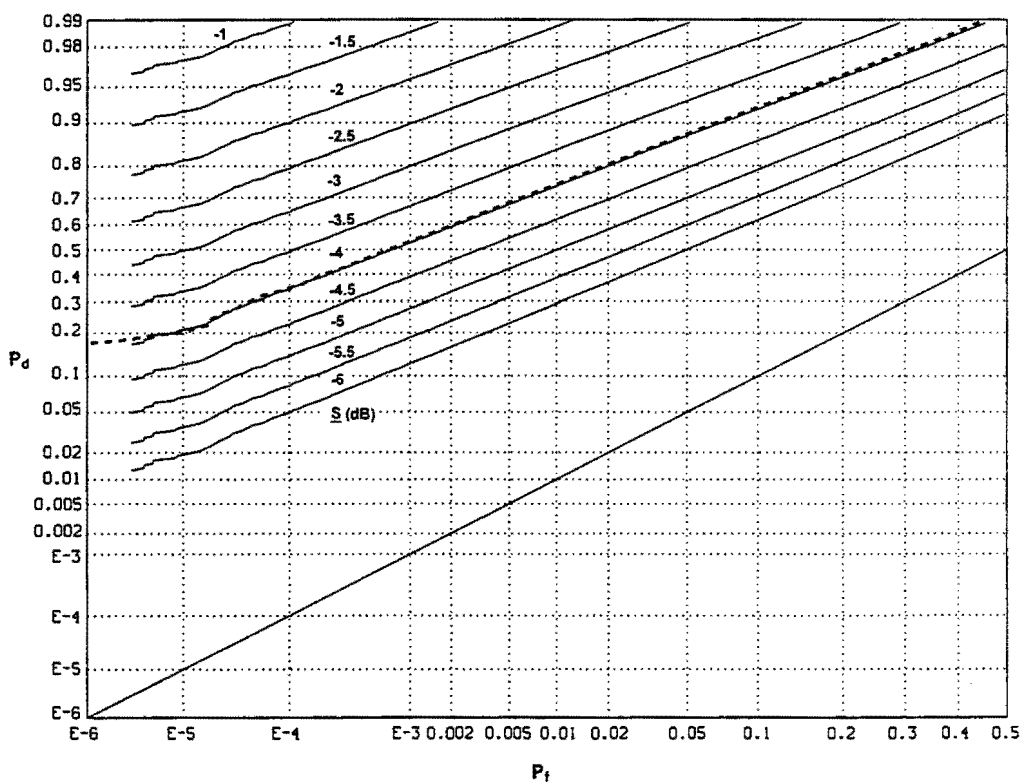
Figure 7. ROC for Maximum Processor with M = 128, N = 1024

23
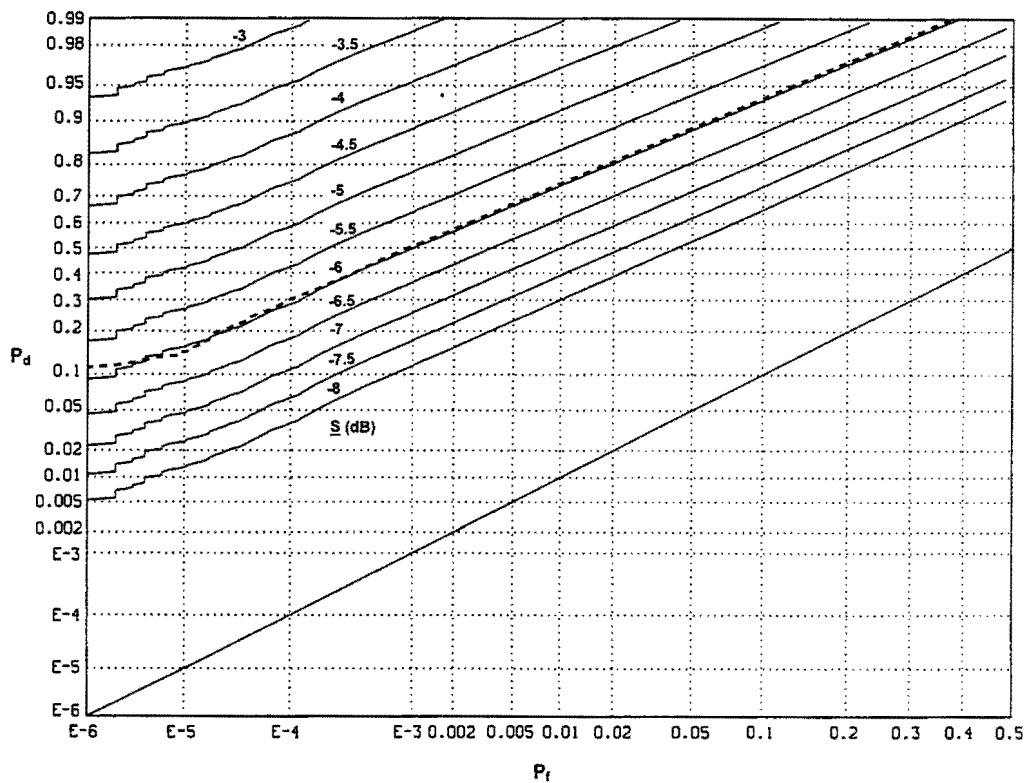
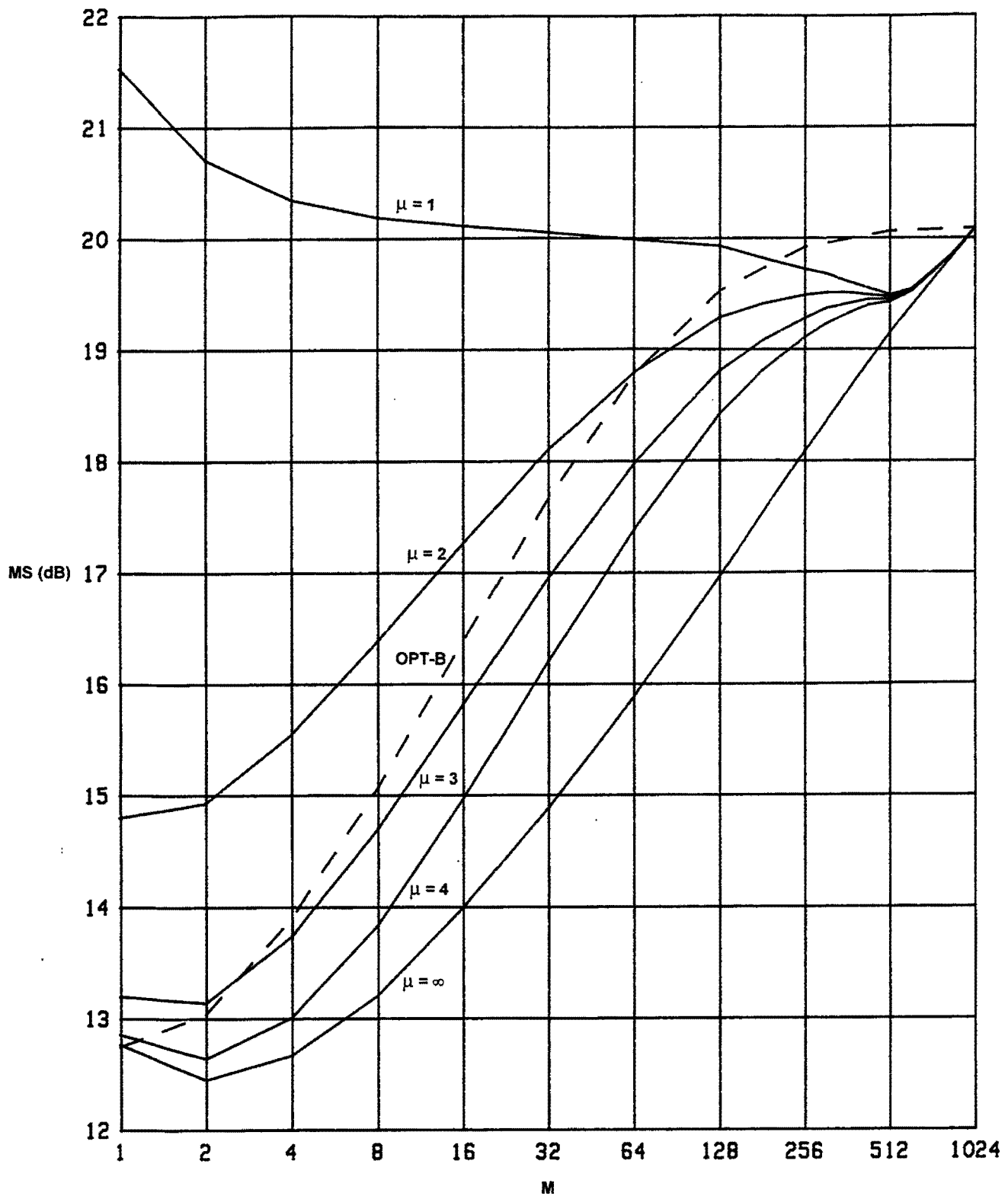Figure 8.   ROC for Maximum Processor with M = 256, N = 1024
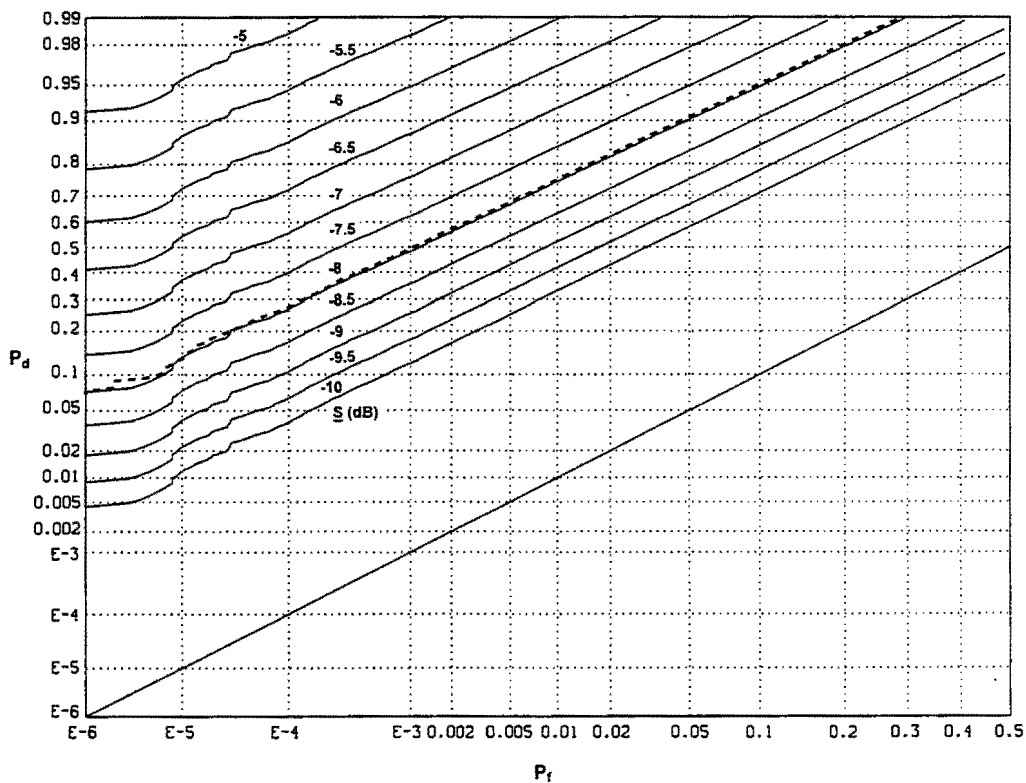
Figure 9.   ROC for Maximum Processor with M = 512, N = 1024

24

Figure 10. Total Signal-to-Noise Ratio Required for $P_f = 0.001$, $P_d = 0.5$, $N = 1024$

SUMMARY

Detection of a random burst signal of known extent M, but of unknown location and signal strength, can be accomplished in a near-optimum fashion by means of the Maximum processor. Preliminary linear sums of duration M, starting at every possible initial bin, are conducted, and the maximum sum is compared with a threshold, for declaration of signal presence or absence. Knowledge of the average received signal strength is not required or used. The discrepancy between the Maximum and Optimum processors is less than 0.1 dB at the standard operating point for search size $N = 1024$ and for signal durations ranging from $M = 1$ to $M = 1024$.

When the signal extent M is also unknown, an additional search over the allowed range of durations must be conducted. Simulation of the optimum (likelihood ratio) processor is possible within a reasonable amount of computer time; however, that numerical study has not been conducted.

REFERENCES

1. A. H. Nuttall, "Detection Performance of a Modified Generalized Likelihood Ratio Processor for Random Signals of Unknown Location," NUWC-NPT Technical Report 10,539, Naval Undersea Warfare Center Detachment, New London, CT, 23 November 1993.

2. A. H. Nuttall, "Detection Performance of Generalized Likelihood Ratio Processors for Random Signals of Unknown Location, Structure, Extent, and Strength," NUWC-NPT Technical Report 10,739, Naval Undersea Warfare Center Detachment, New London, CT, 25 August 1994.

3. A. H. Nuttall, "Detection Performance of Power-Law Processors for Random Signals of Unknown Location, Structure, Extent, and Strength," NUWC-NPT Technical Report 10,751, Naval Undersea Warfare Center Detachment, New London, CT, 16 September 1994.

4. A. H. Nuttall, "Limiting Detection Performance for Random Signals of Unknown Location, Structure, Extent, and Strength," NUWC-NPT Technical Report 10,839, Naval Undersea Warfare Center Detachment, New London, CT, 2 March 1995.

5. A. H. Nuttall, "Near-Optimum Detection Performance of Power-Law Processors for Random Signals of Unknown Locations, Structure, Extent, and Arbitrary Strengths," NUWC-NPT Technical Report 11,123, Naval Undersea Warfare Center Detachment, New London, CT, 15 April 1996.

6. A. H. Nuttall, "Performance of Power-Law Processor with Normalization for Random Signals of Unknown Structure," NUWC-NPT Technical Report 10,760, Naval Undersea Warfare Center Division, Newport, RI, 5 May 1997.

APPENDIX
OPERATING CHARACTERISTICS FOR LAP PROCESSOR

The LAP (linear and power) processor is characterized by test

$$\sum_{m=1}^{N+1-M} \left(X_M(m) \right)^{\mu} \underset{<}{>} v , \quad \text{with} \quad X_M(m) = \sum_{n=m}^{M-1+m} x_n . \qquad (A-1)$$

The LAP receiver operating characteristics for powers μ = 1, 2, 3, 4 and signal durations M = 2, 4, 8, 16, 32, 64, 128, 256, 512 are collected in this appendix; the search region is fixed at size N = 1024 in all cases. The ROCs are based on one million independent trials of test (A-1); thus, the smaller false alarm probability estimates P_f are not reliable.

It will be observed from these characteristics that the performance of LAP processor (A-1) improves monotonically with power μ, regardless of the value of M; in fact, the Maximum processor (μ = ∞) is virtually optimum, as was shown in figures 1 through 10 of the main text. The total signal-to-noise ratios in figure 10 for μ = 1, 2, 3, 4, required to realize the standard operating point P_f = 0.001, P_d = 0.5, were extracted from figures A-1 through A-36 here. Other higher quality operating points can also be investigated from the figures in this appendix.
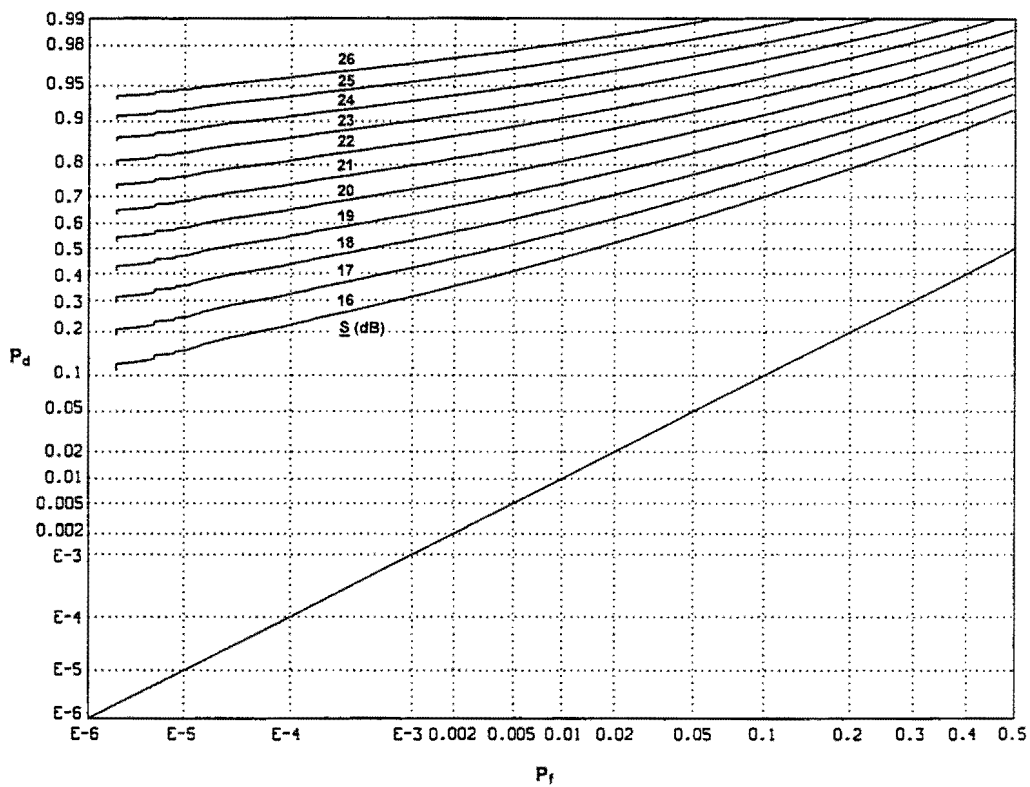
Figure A-1.   ROC for LAP Processor with $\mu = 1$, $M = 2$, $N = 1024$
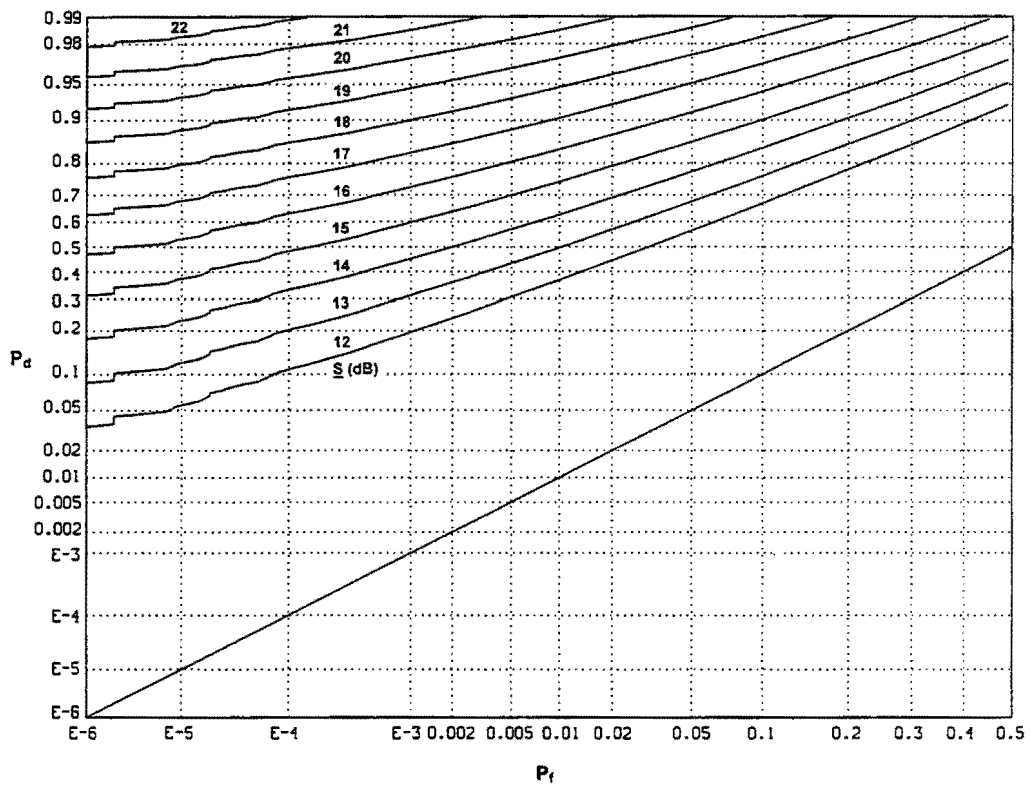
Figure A-2.   ROC for LAP Processor with $\mu = 1$, $M = 4$, $N = 1024$
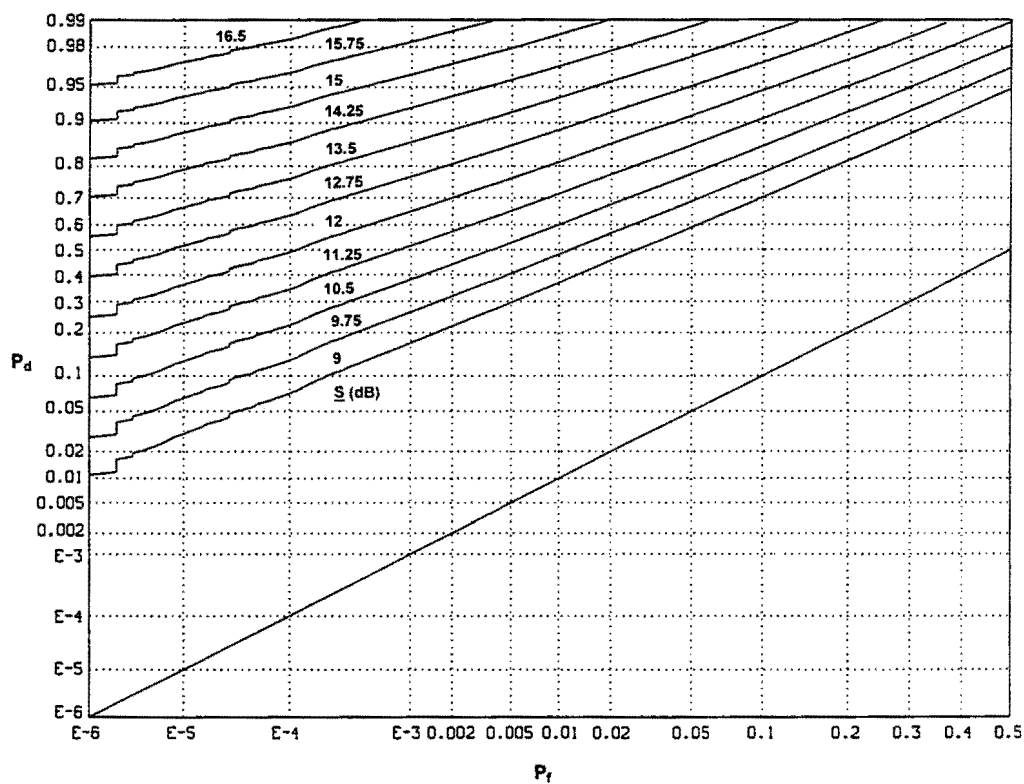
Figure A-3.   ROC for LAP Processor with $\mu = 1$, $M = 8$, $N = 1024$
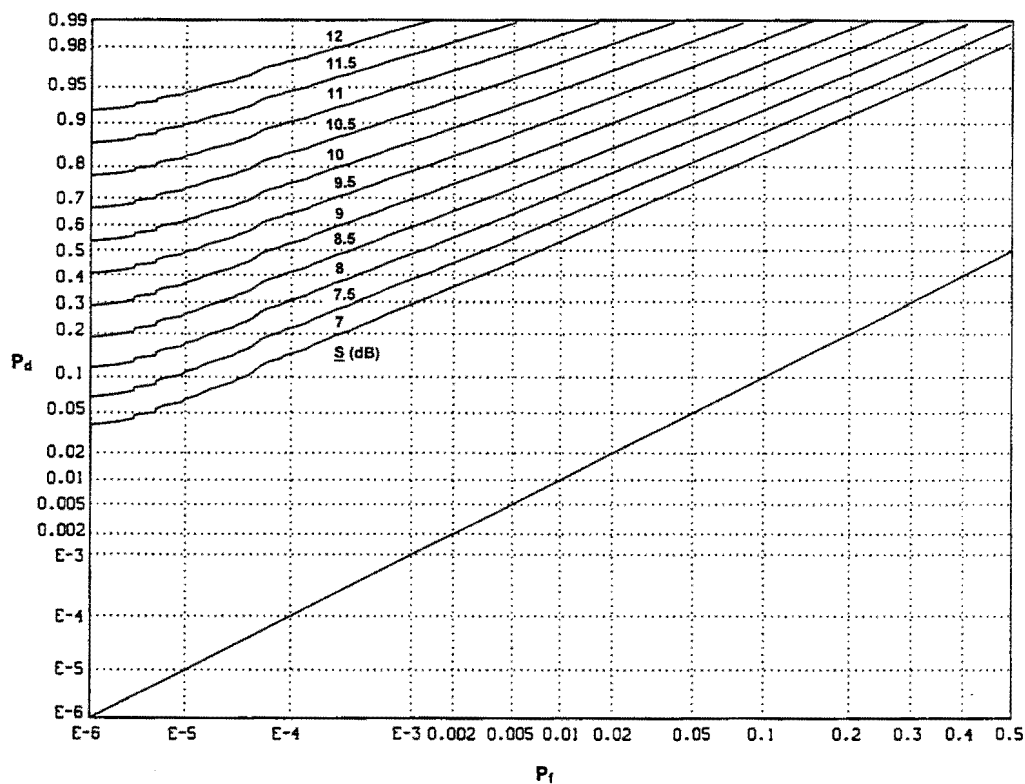

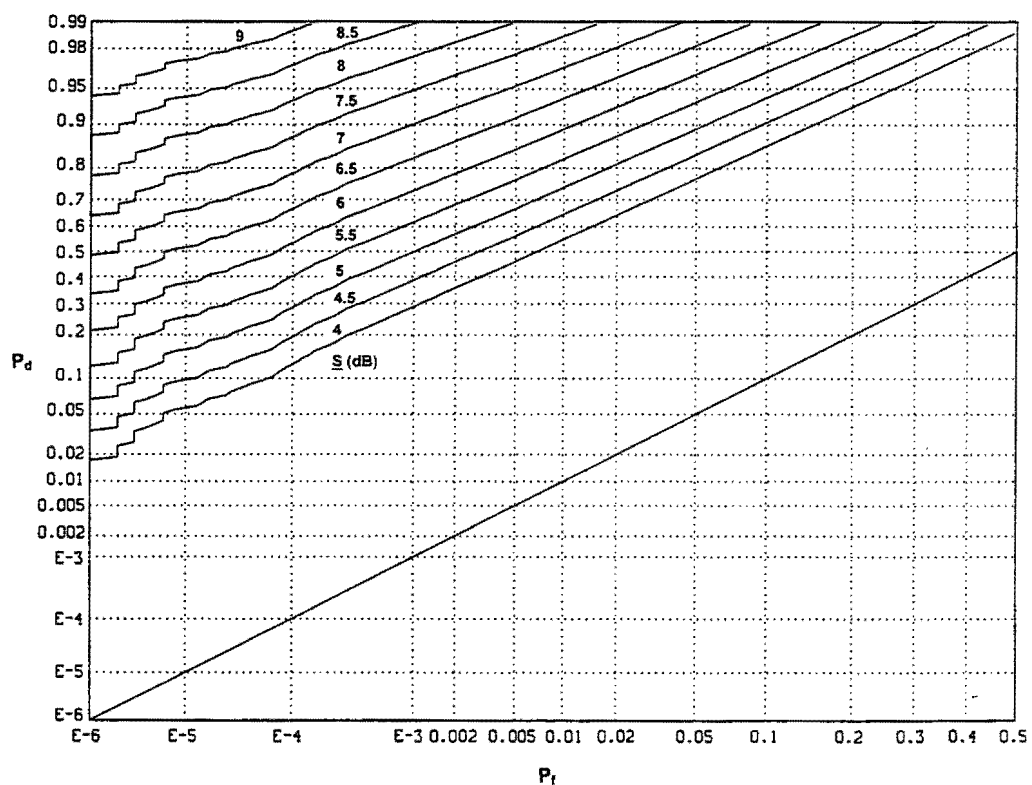Figure A-4.   ROC for LAP Processor with $\mu = 1$, $M = 16$, $N = 1024$

Figure A-5. ROC for LAP Processor with $\mu = 1$, $M = 32$, $N = 1024$
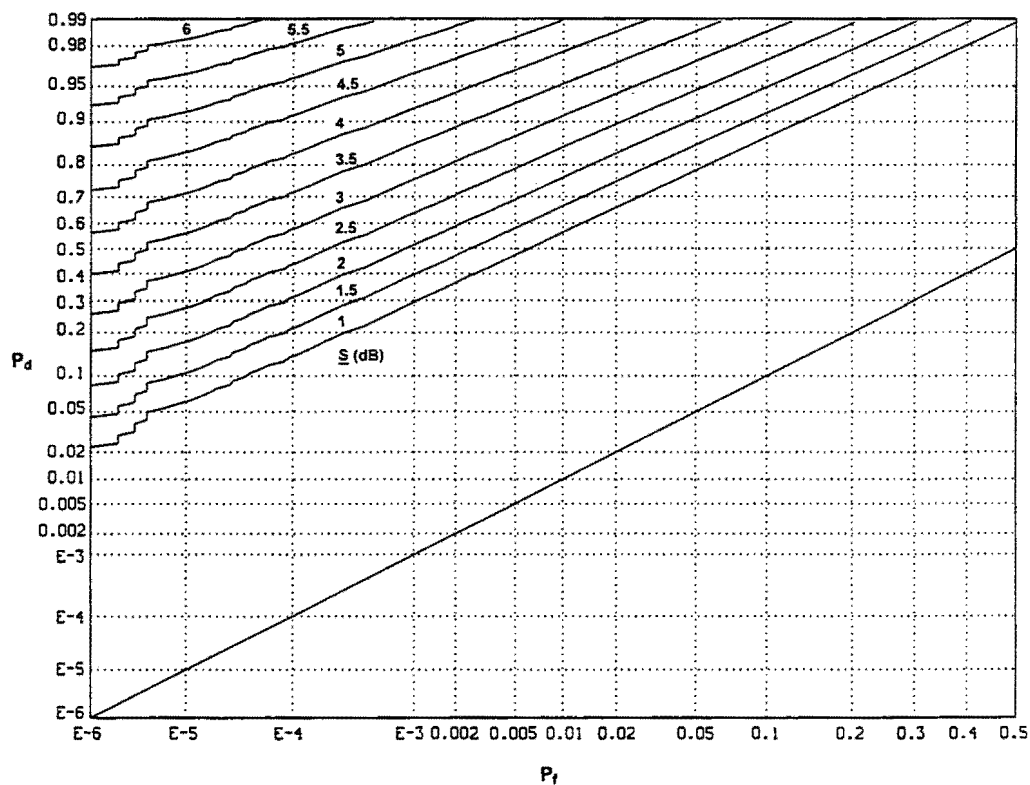

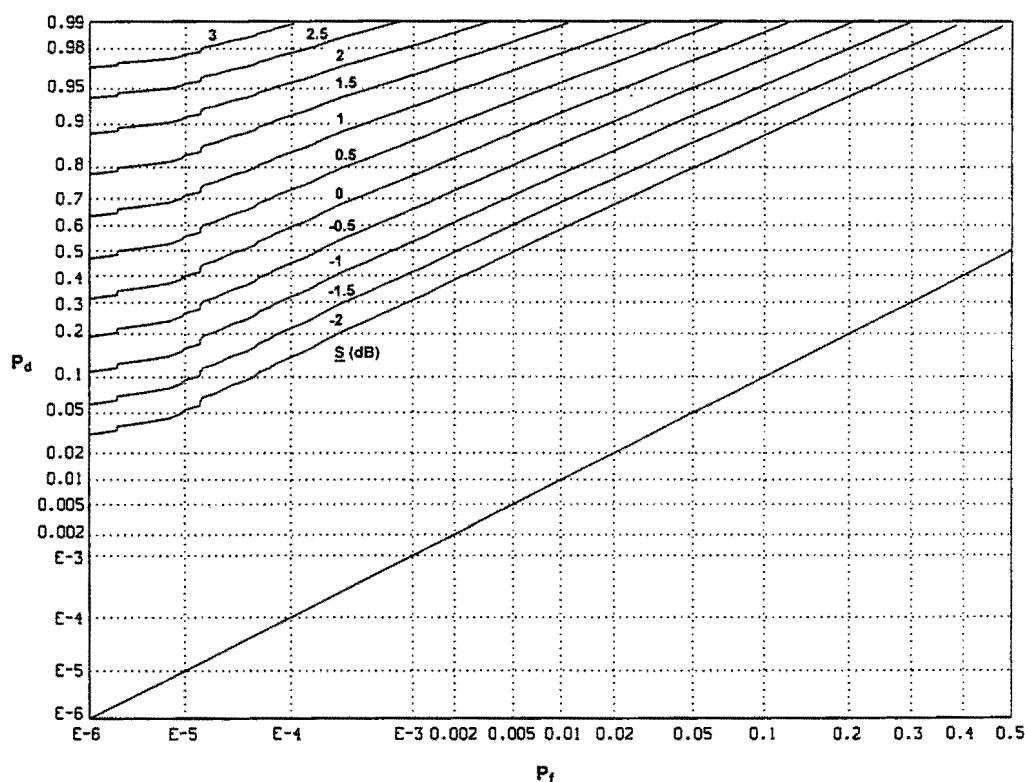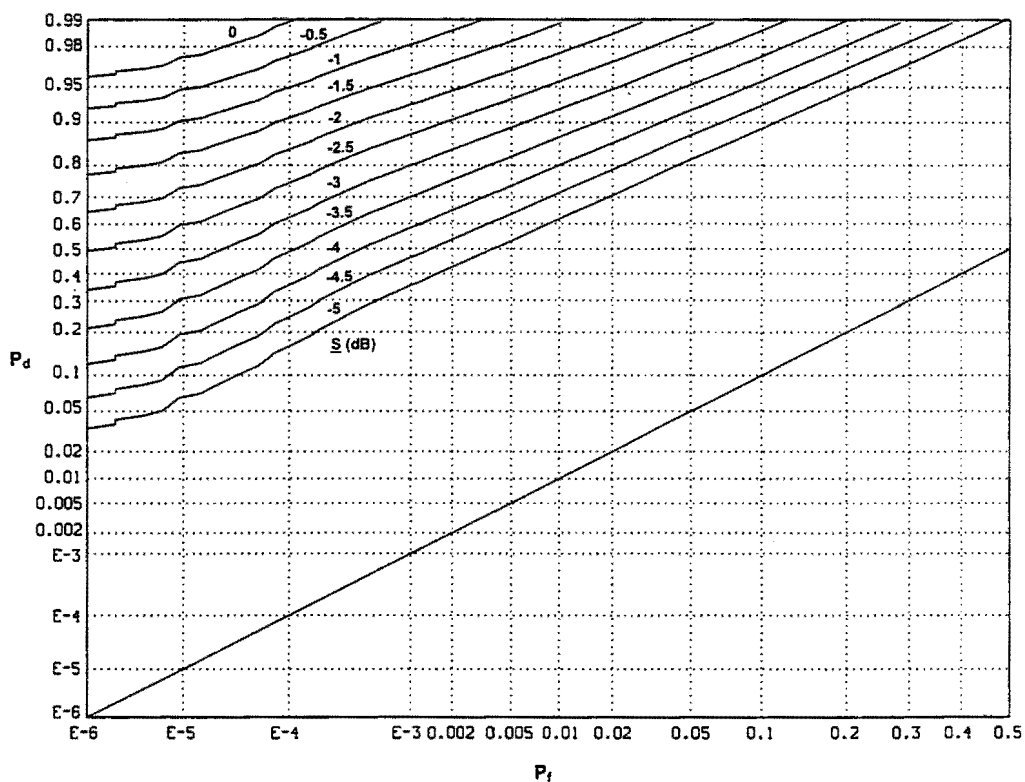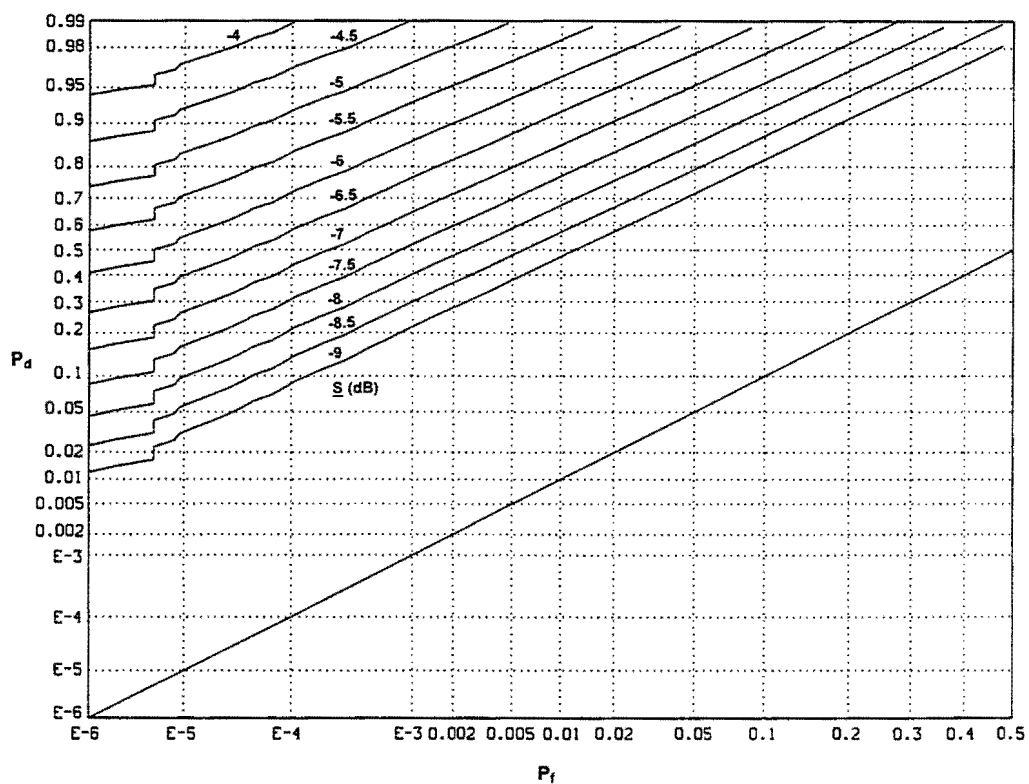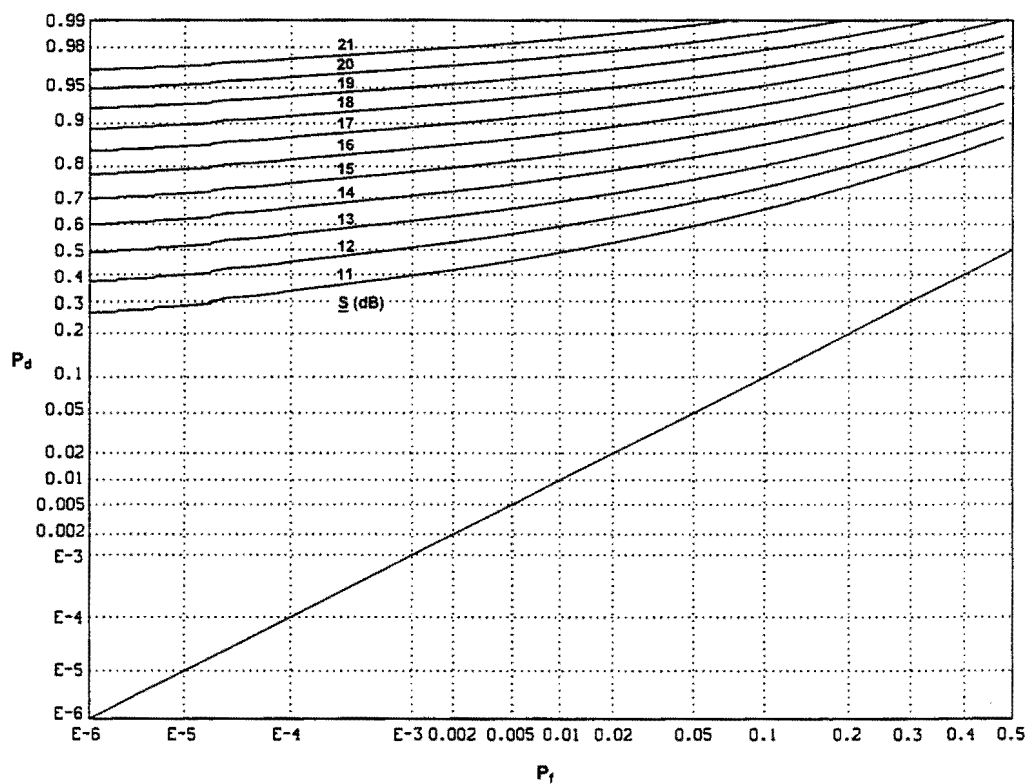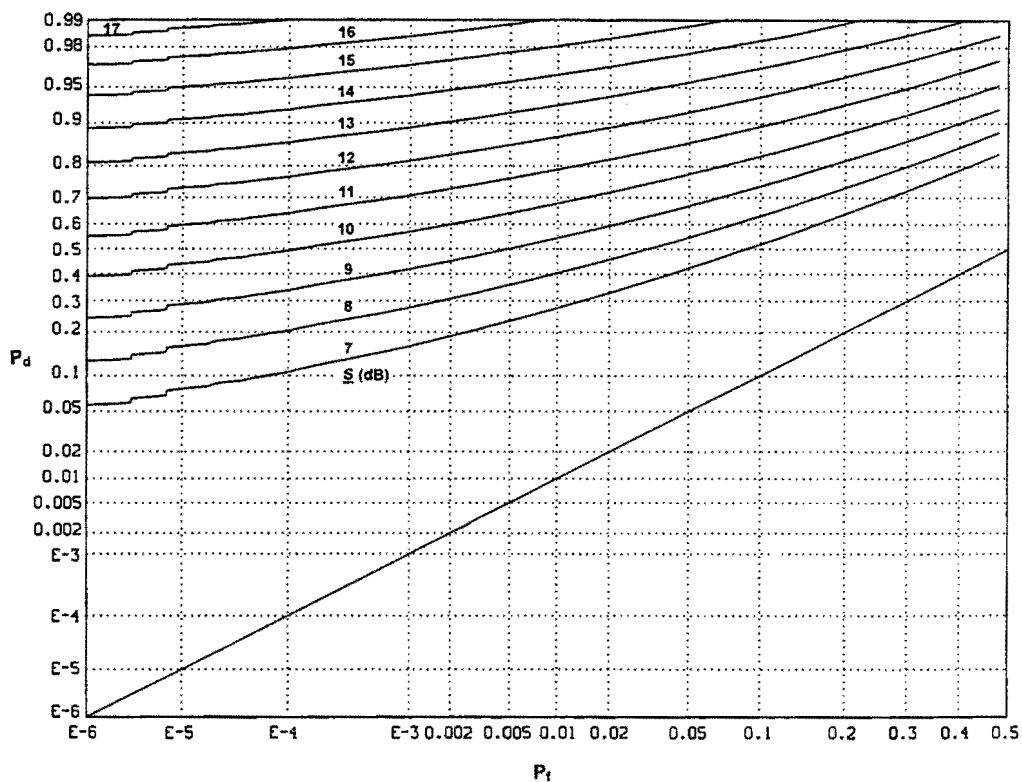Figure A-6. ROC for LAP Processor with $\mu = 1$, $M = 64$, $N = 1024$
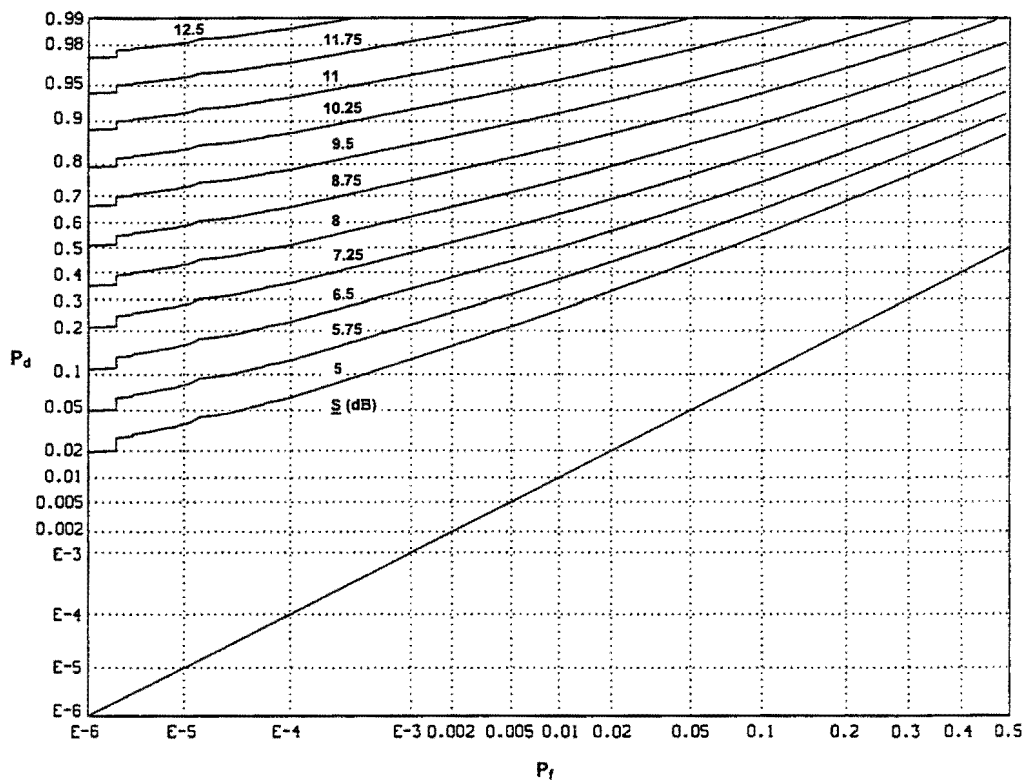
Figure A-7.   ROC for LAP Processor with $\mu = 1$, M = 128, N = 1024

Figure A-8.   ROC for LAP Processor with $\mu = 1$, M = 256, N = 1024

Figure A-9.   ROC for LAP Processor with $\mu = 1$, M = 512, N = 1024

Figure A-10.   ROC for LAP Processor with $\mu = 2$, M = 2, N = 1024

A-6

Figure A-11. ROC for LAP Processor with $\mu = 2$, M = 4, N = 1024

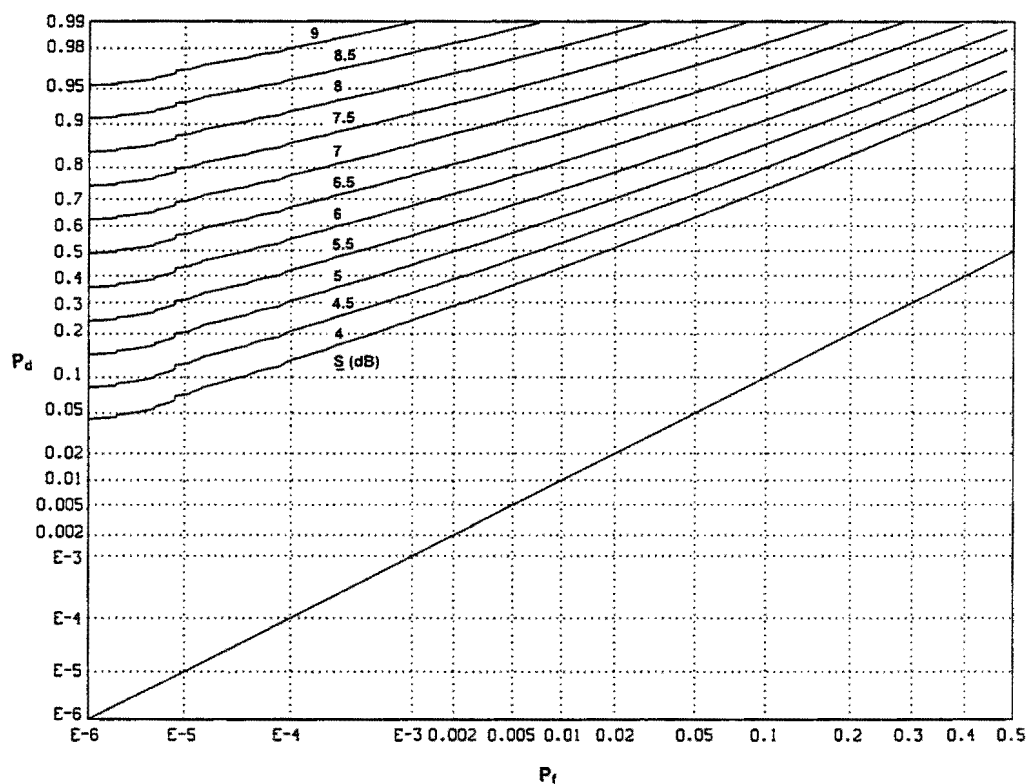Figure A-12. ROC for LAP Processor with $\mu = 2$, M = 8, N = 1024

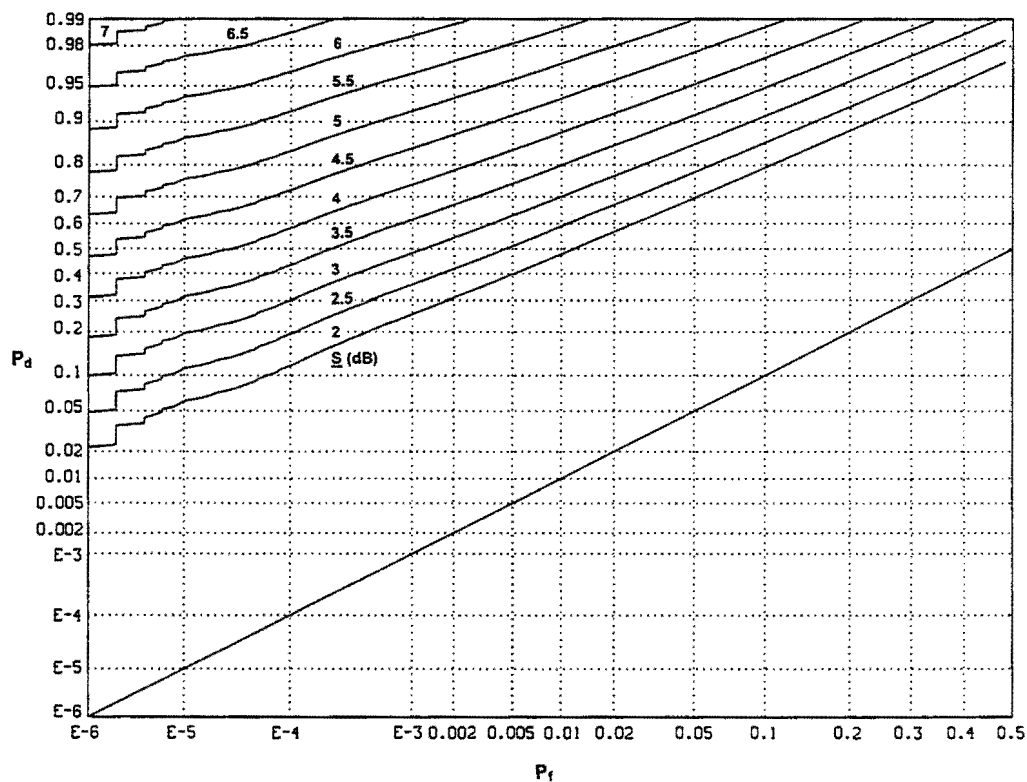Figure A-13. ROC for LAP Processor with $\mu = 2$, $M = 16$, $N = 1024$

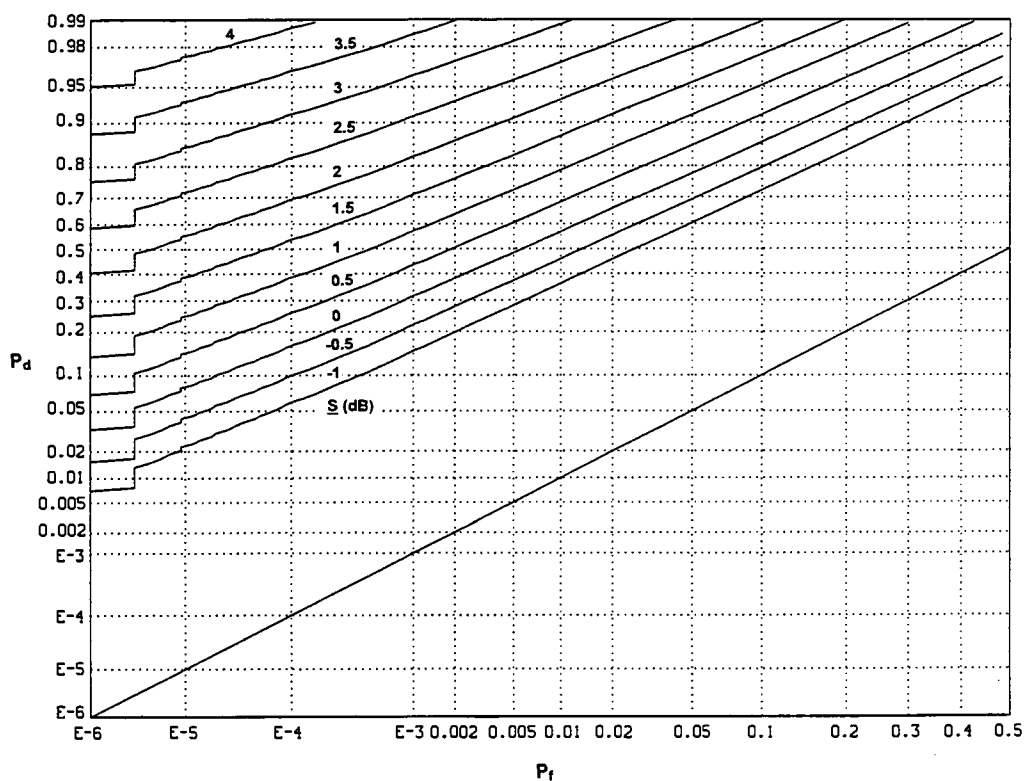Figure A-14. ROC for LAP Processor with $\mu = 2$, $M = 32$, $N = 1024$

A-8

Figure A-15. ROC for LAP Processor with $\mu = 2$, M $= 64$, N $= 1024$

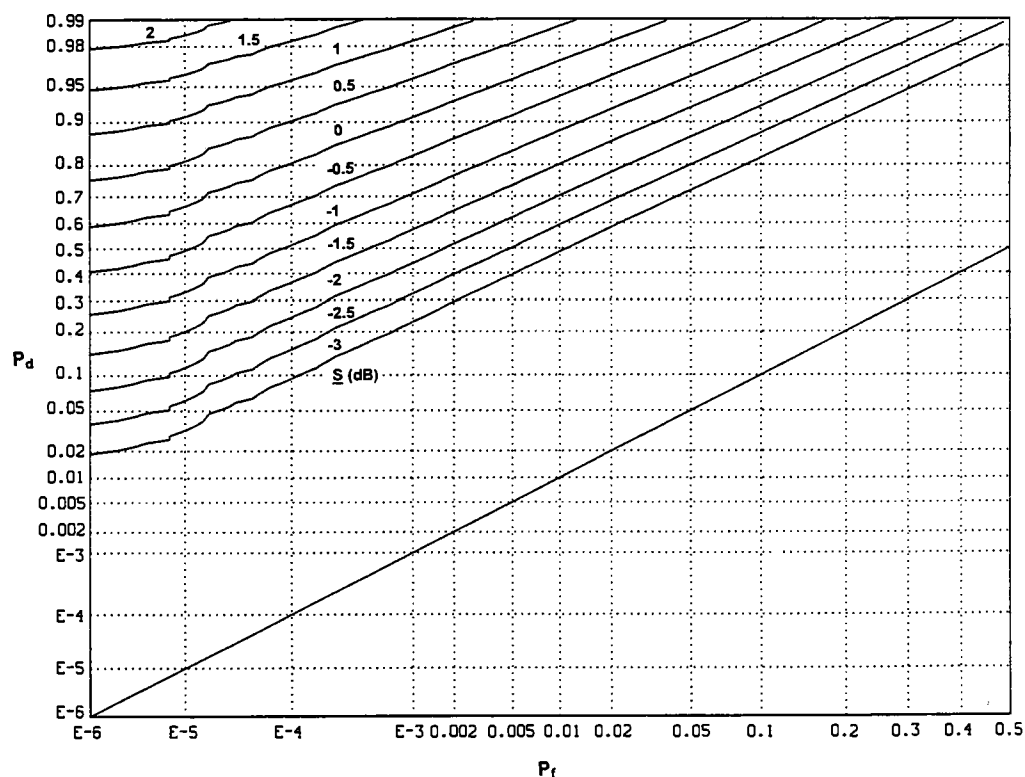**Figure A-16.** ROC for LAP Processor with $\mu = 2$, M $= 128$, N $= 1024$

Figure A-17.   ROC for LAP Processor with $\mu = 2$, M = 256, N = 1024

Figure A-18.   ROC for LAP Processor with $\mu = 2$, M = 512, N = 1024

Figure A-19.   ROC for LAP Processor with $\mu = 3$, $M = 2$, $N = 1024$

Figure A-20.   ROC for LAP Processor with $\mu = 3$, $M = 4$, $N = 1024$

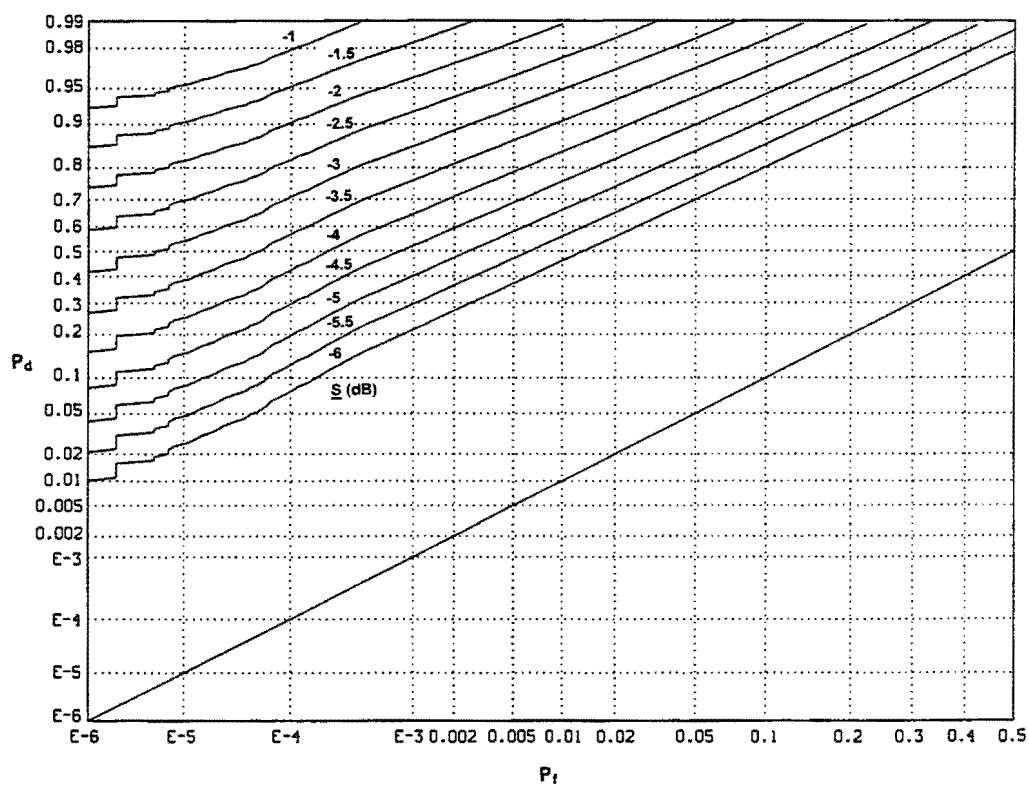Figure A-21. ROC for LAP Processor with $\mu = 3$, $M = 8$, $N = 1024$

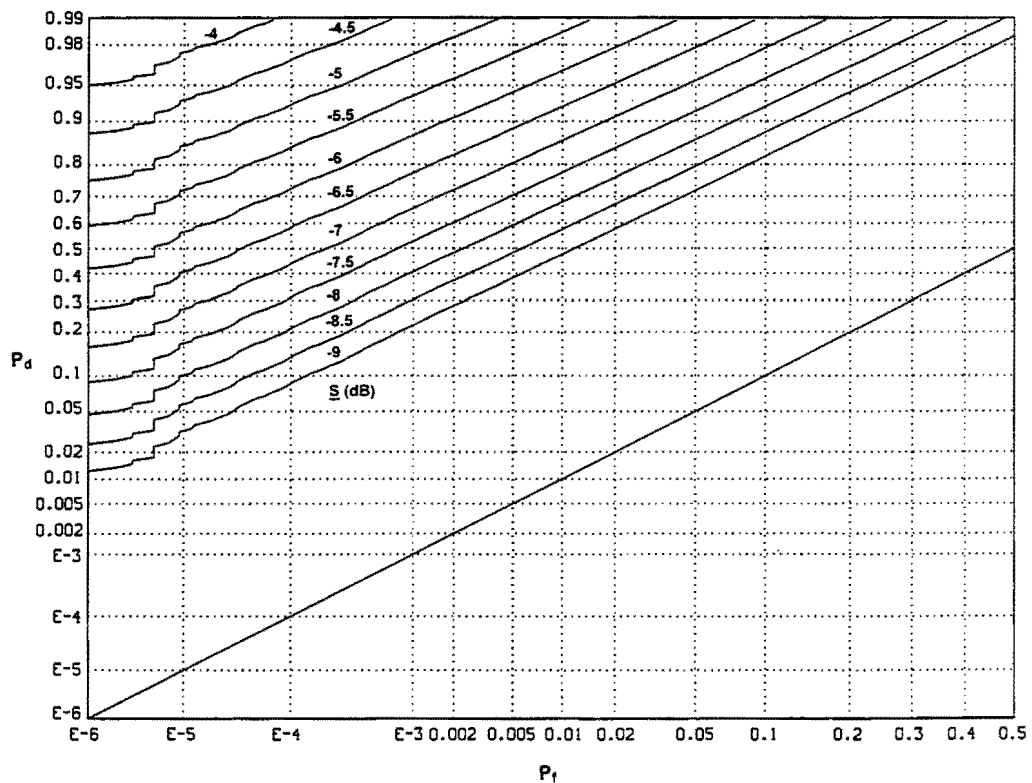Figure A-22. ROC for LAP Processor with $\mu = 3$, $M = 16$, $N = 1024$
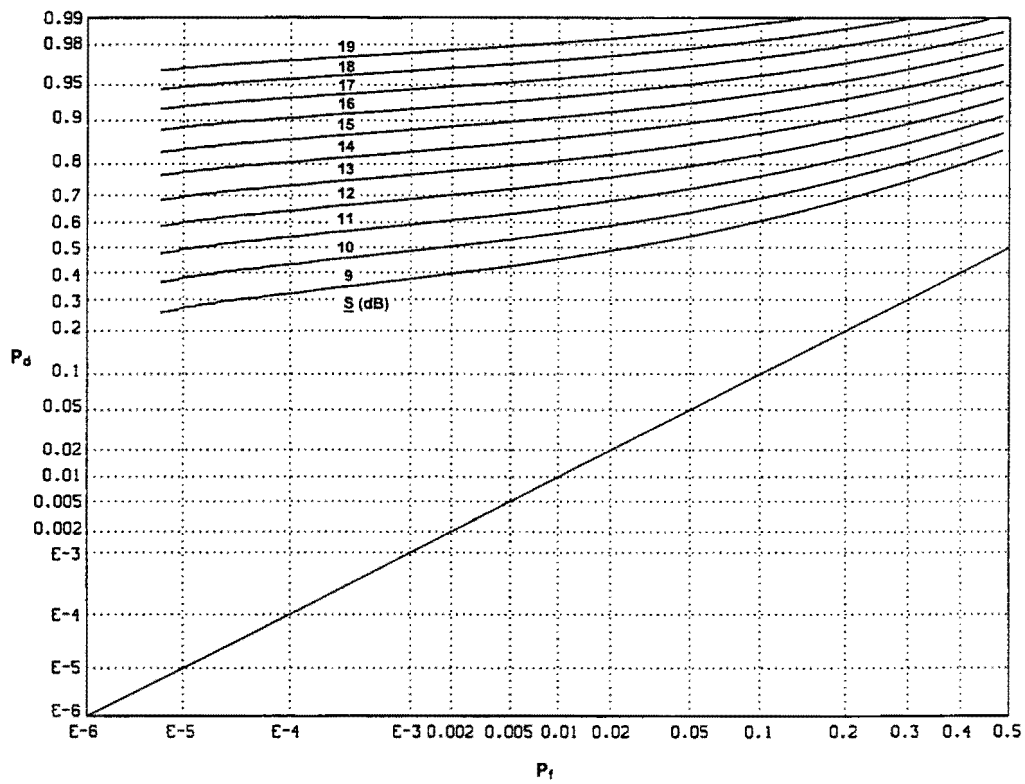
A-12

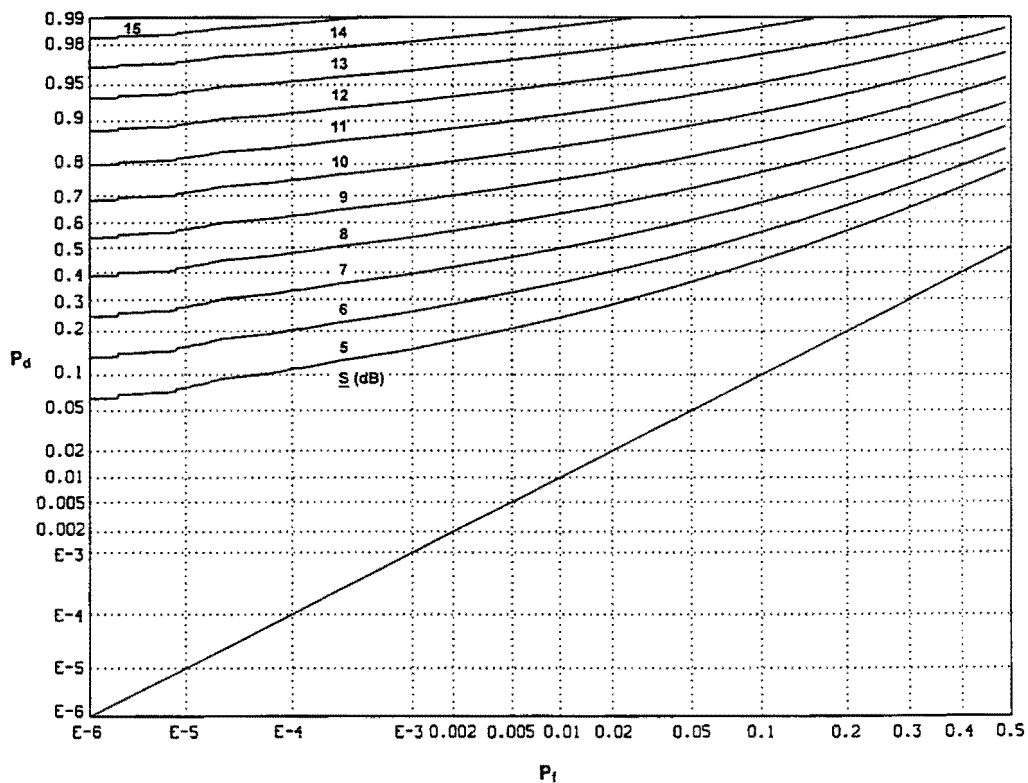Figure A-23. ROC for LAP Processor with $\mu = 3$, $M = 32$, $N = 1024$

Figure A-24. ROC for LAP Processor with $\mu = 3$, $M = 64$, $N = 1024$

Figure A-25.   ROC for LAP Processor with $\mu = 3$, $M = 128$, $N = 1024$

Figure A-26.   ROC for LAP Processor with $\mu = 3$, $M = 256$, $N = 1024$

A-14

Figure A-27.   ROC for LAP Processor with $\mu = 3$, M = 512, N = 1024

Figure A-28.   ROC for LAP Processor with $\mu = 4$, M = 2, N = 1024

A-15

Figure A-29.   ROC for LAP Processor with $\mu = 4$, $M = 4$, $N = 1024$

Figure A-30.   ROC for LAP Processor with $\mu = 4$, $M = 8$, $N = 1024$

A-16

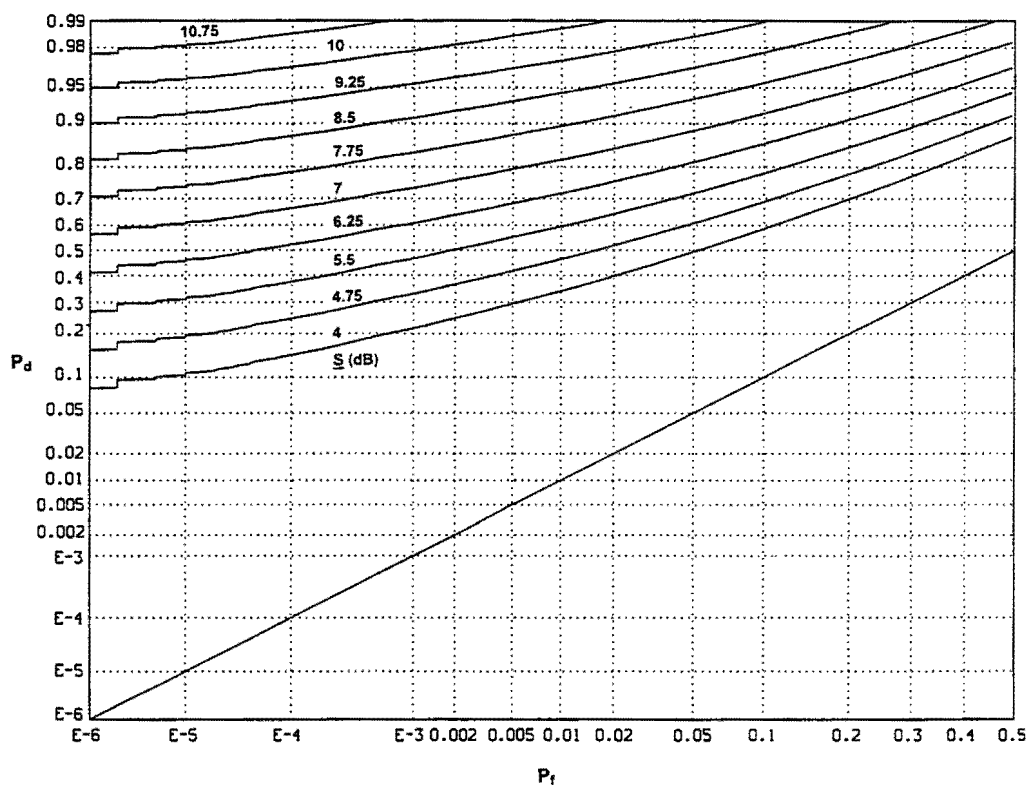**Figure A-31.** ROC for LAP Processor with $\mu = 4$, $M = 16$, $N = 1024$
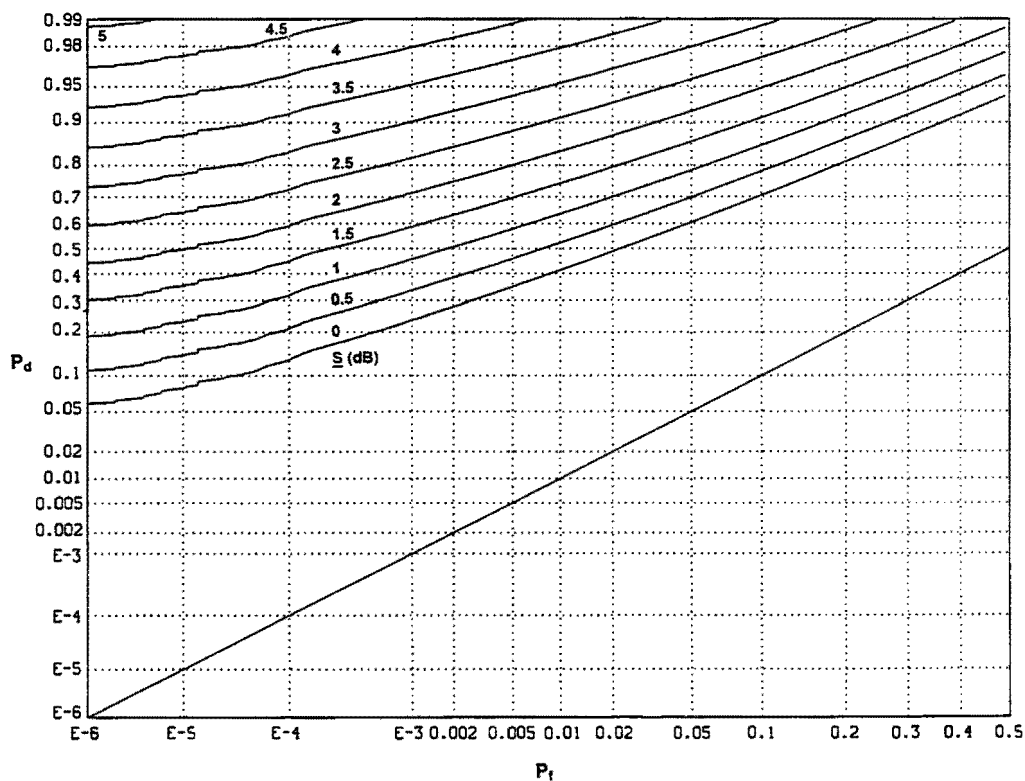
Figure A-32. ROC for LAP Processor with $\mu = 4$, $M = 32$, $N = 1024$
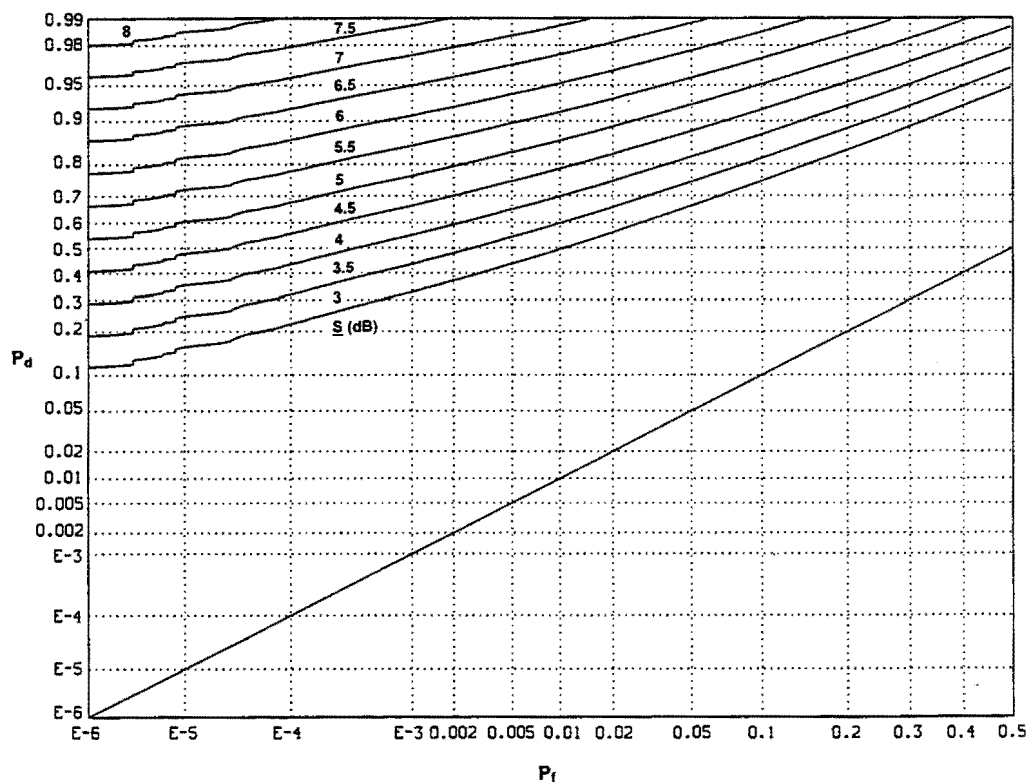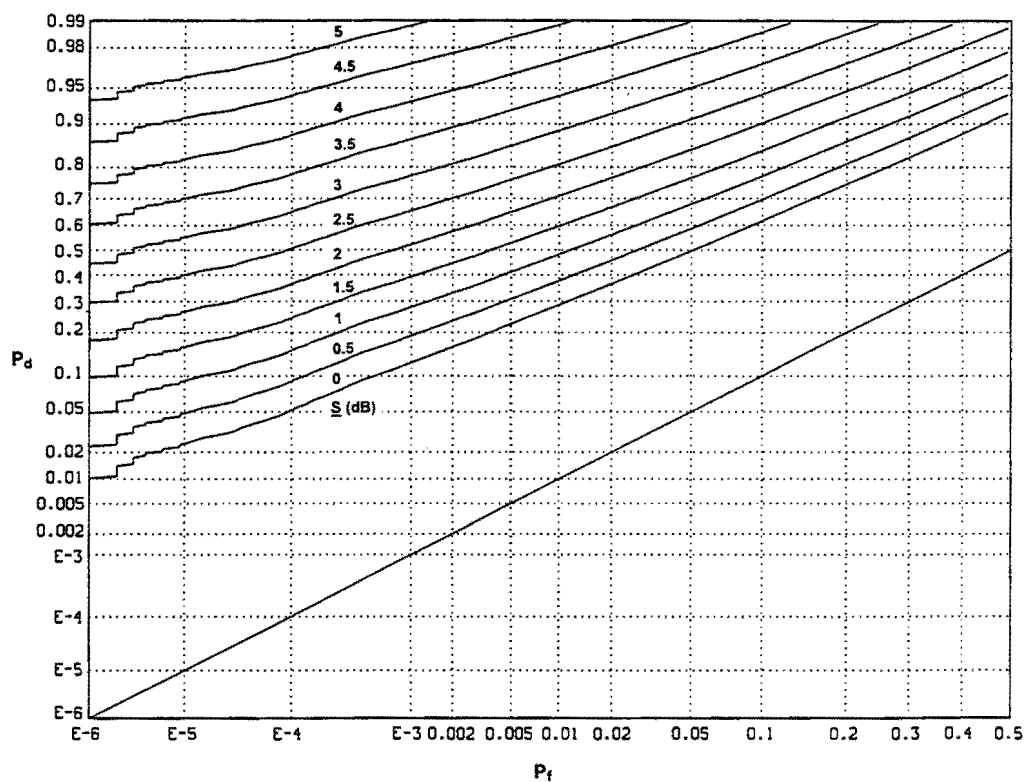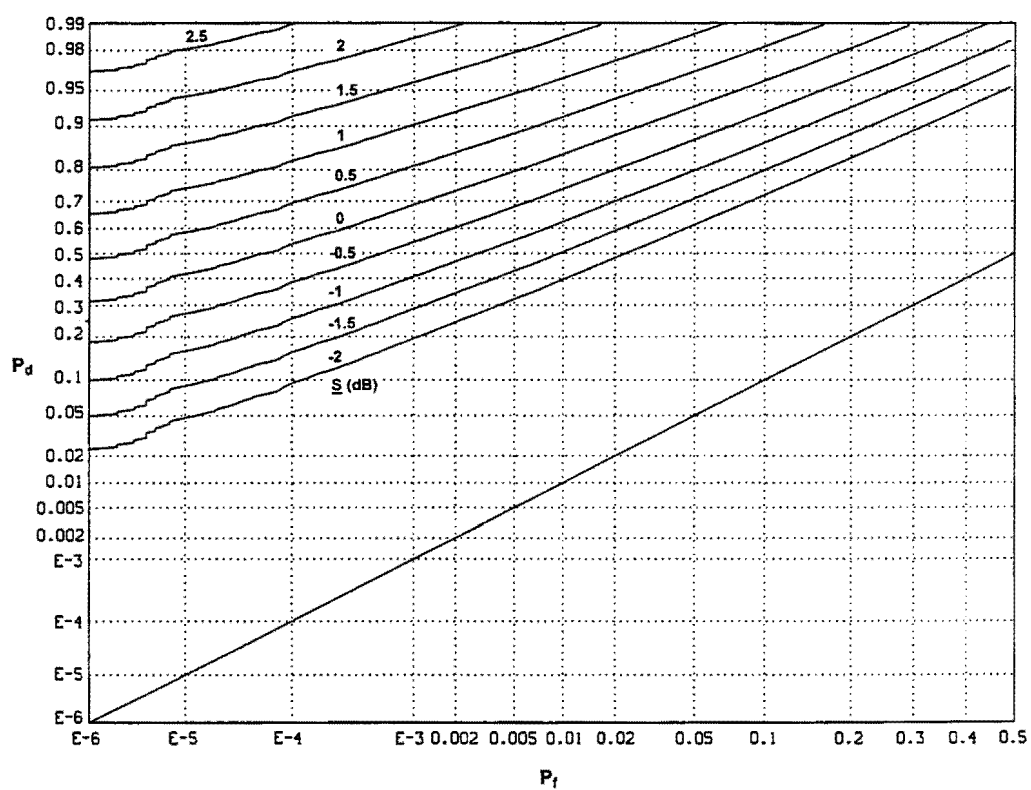
Figure A-33.   ROC for LAP Processor with $\mu = 4$, M = 64, N = 1024
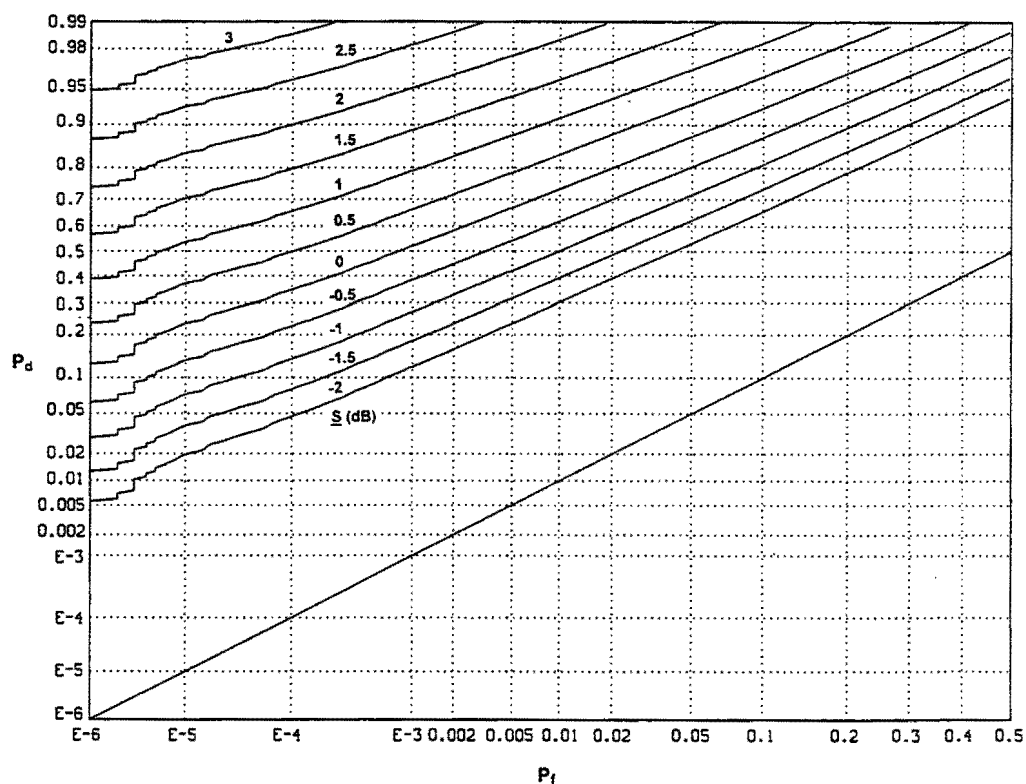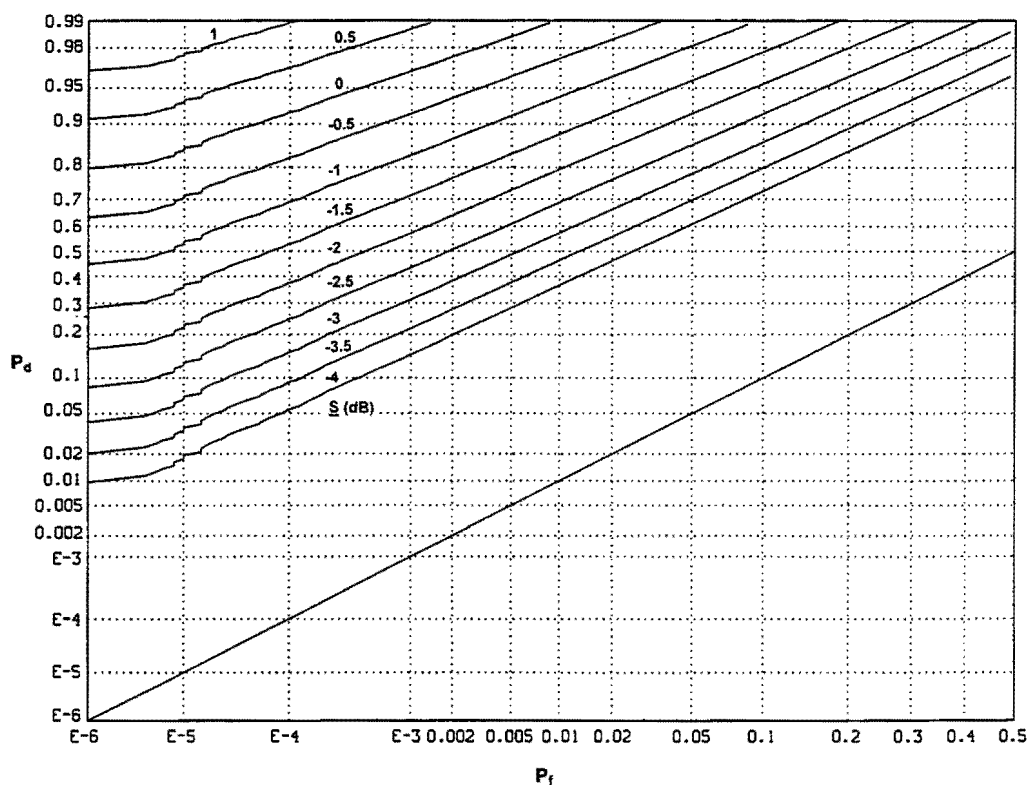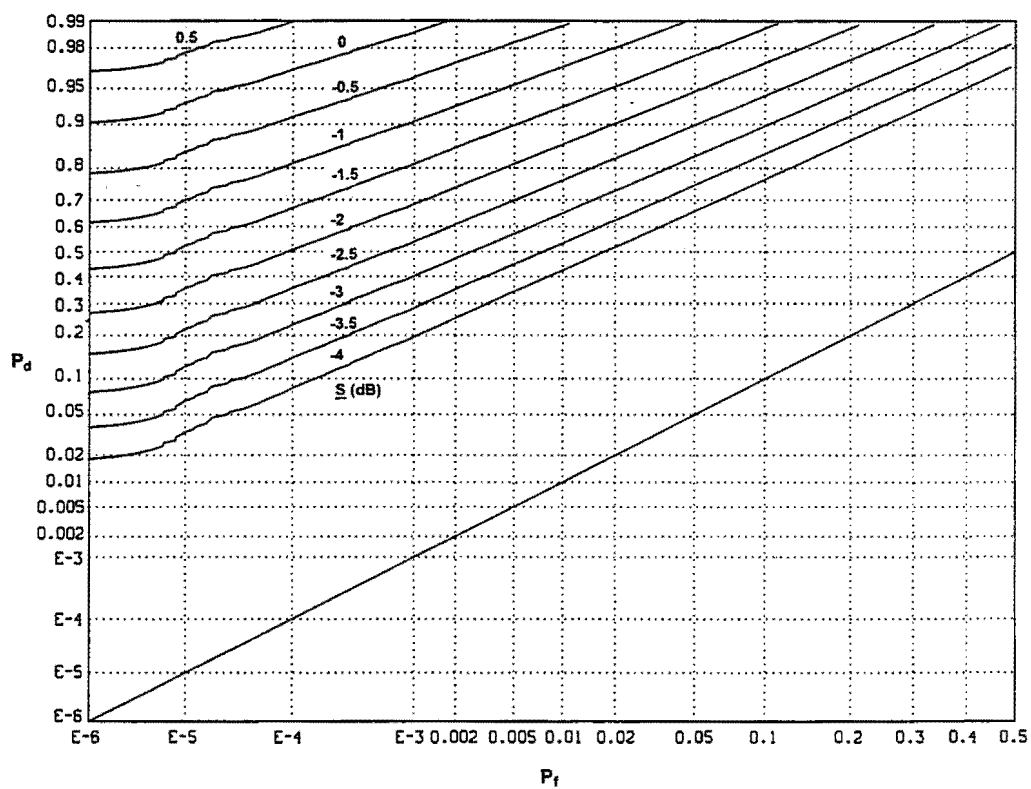

Figure A-34.   ROC for LAP Processor with $\mu = 4$, M = 128, N = 1024
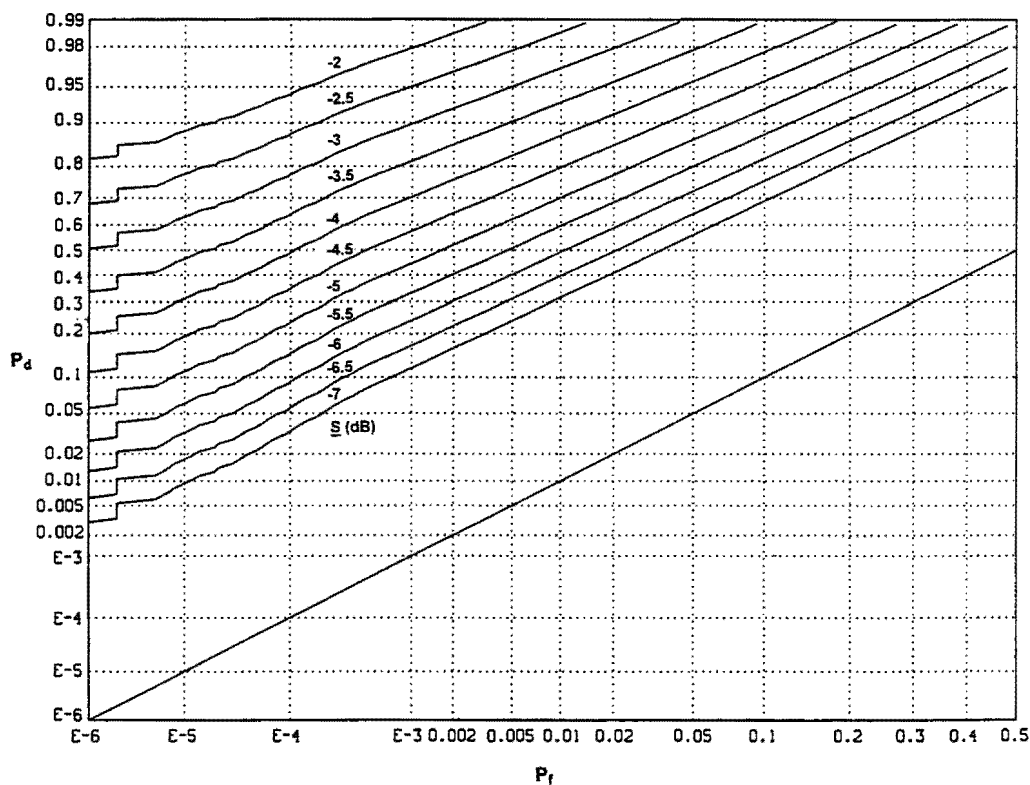
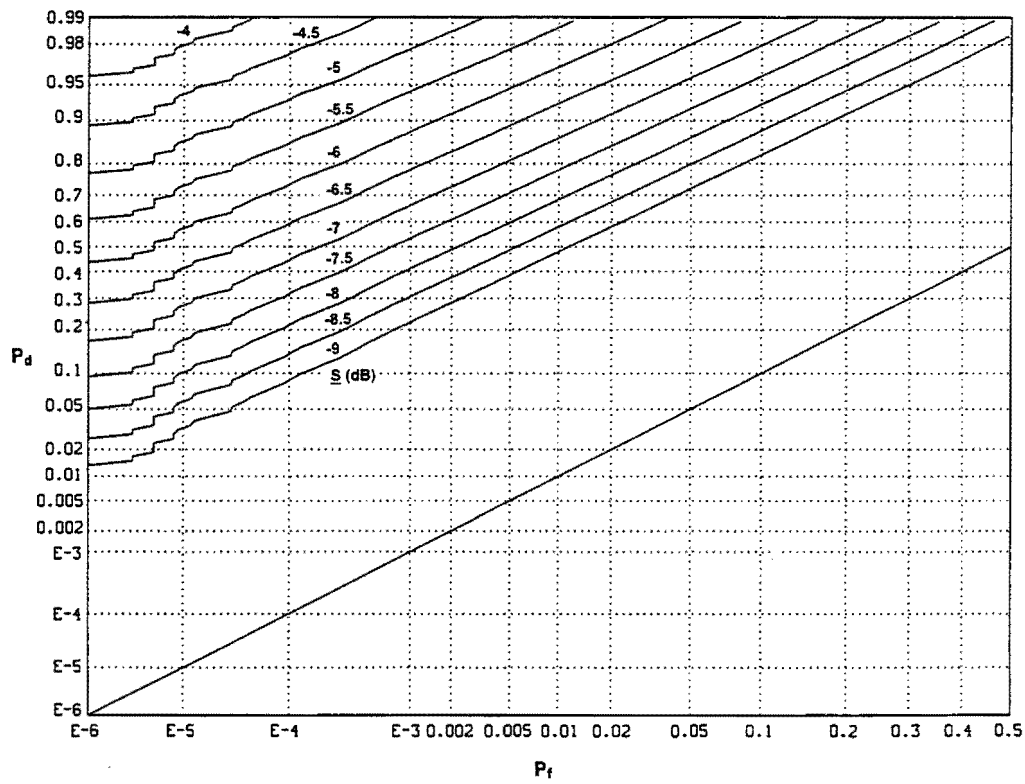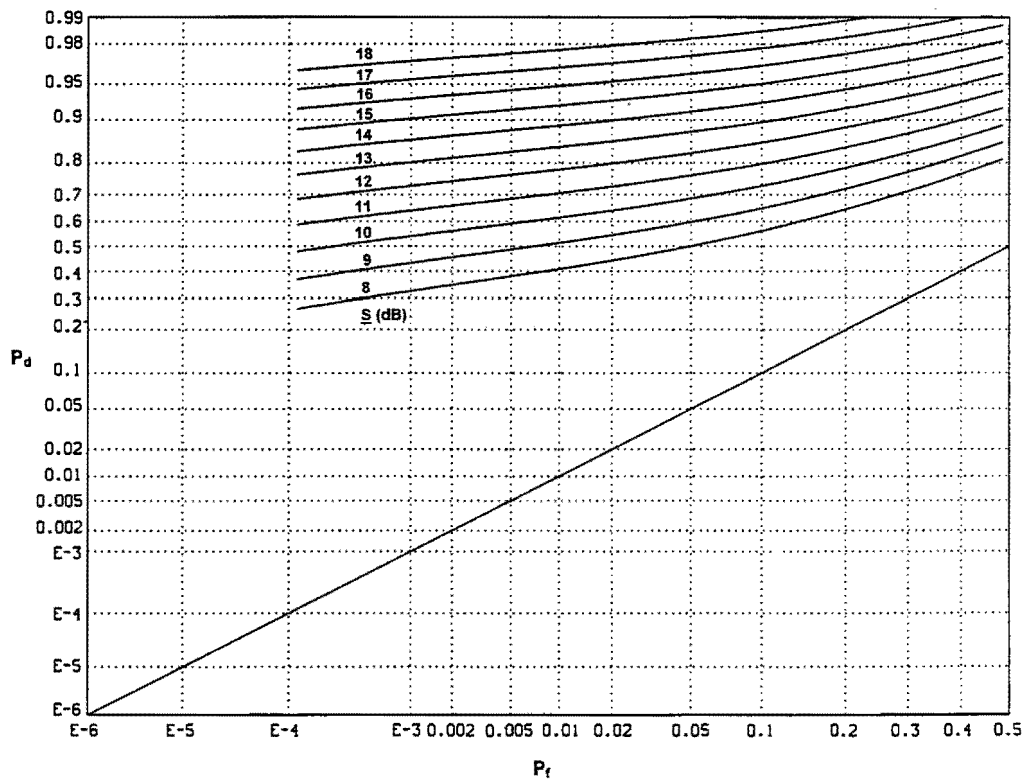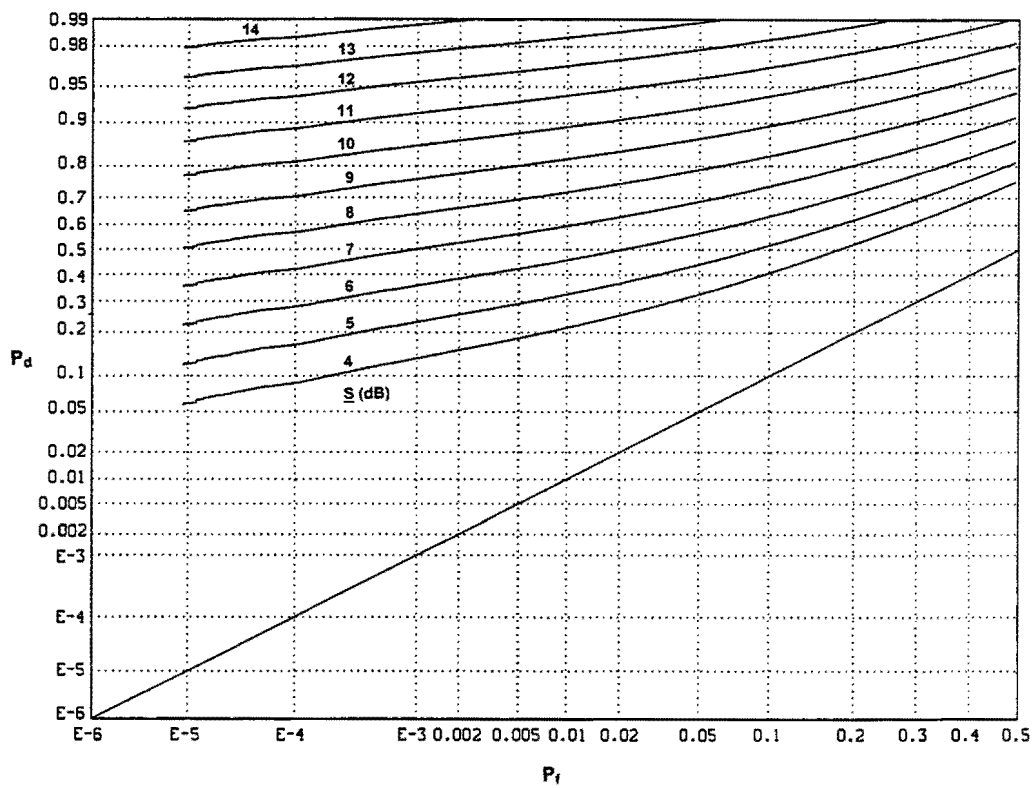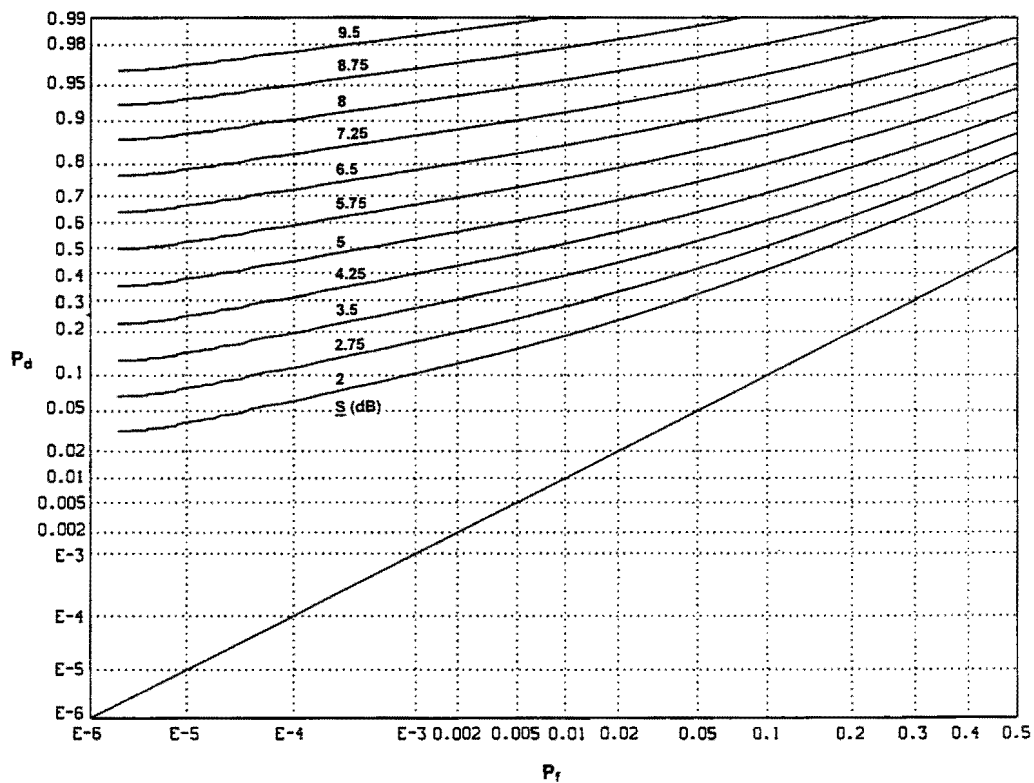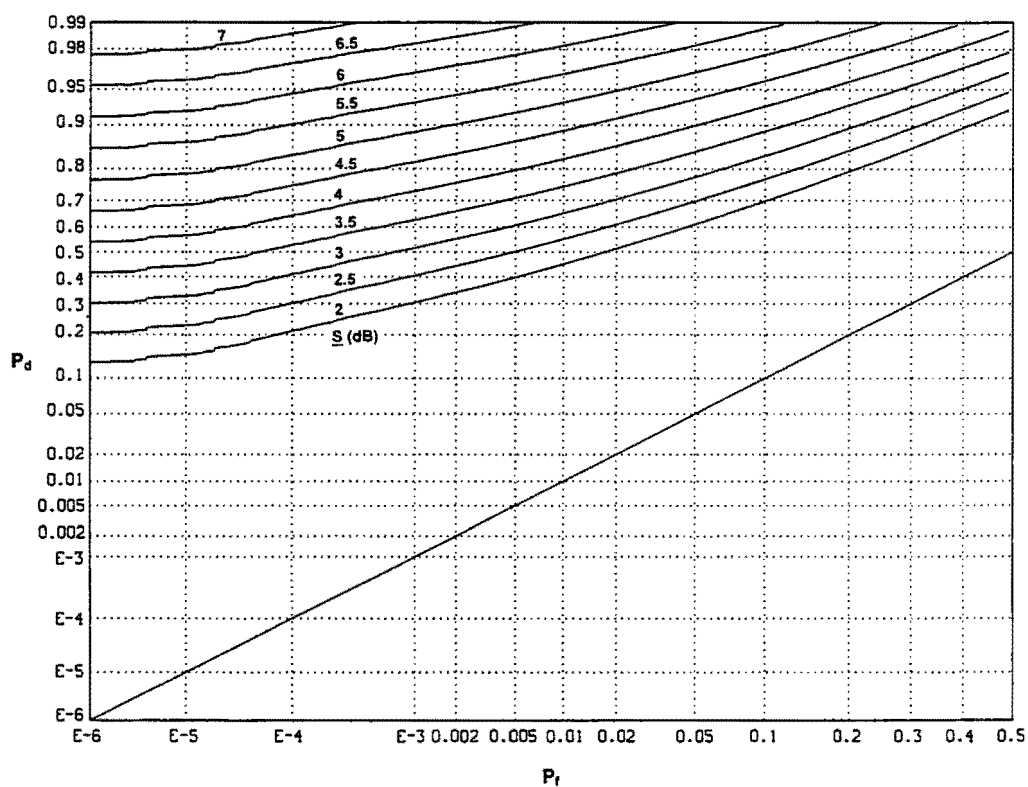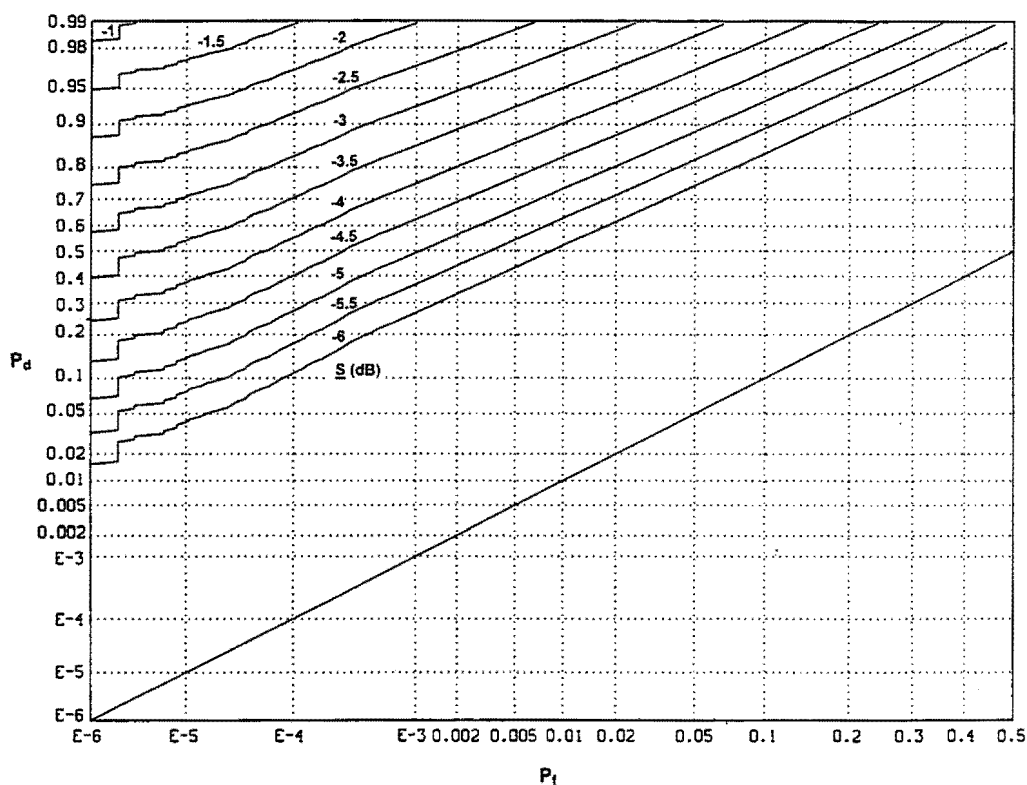Figure A-35. ROC for LAP Processor with $\mu = 4$, M = 256, N = 1024
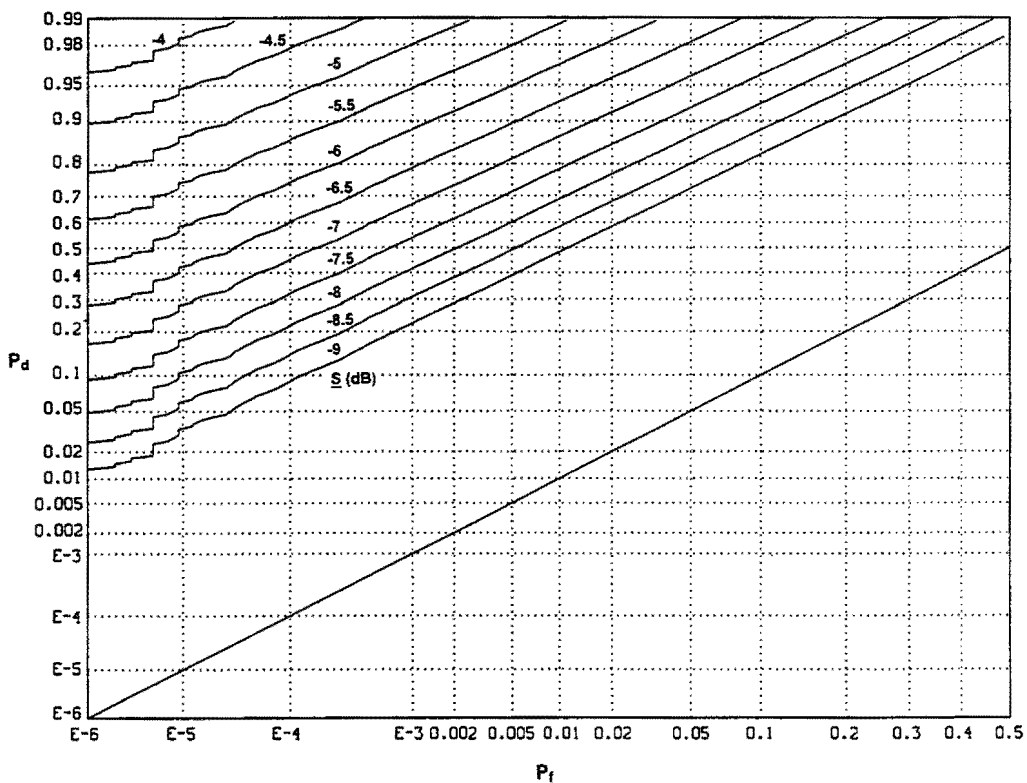
Figure A-36. ROC for LAP Processor with $\mu = 4$, M = 512, N = 1024

INITIAL DISTRIBUTION LIST

Addressee	No. of Copies
Center for Naval Analyses, VA	1
Coast Guard Academy, CT	
J. Wolcin	1
Defense Technical Information Center, VA	12
Griffiss Air Force Base, NY	
Documents Library	1
J. Michels	1
Hanscom Air Force Base, MA	
M. Rangaswamy	1
National Radio Astronomy Observatory, VA	
F. Schwab	1
National Security Agency, MD	
J. Maar	1
National Technical Information Service, VA	10
Naval Air Warfare Center, PA	
L. Allen	1
Naval Command Control and Ocean Surveillance Center, CA	
J. Alsup	1
W. Marsh	1
P. Nachtigall	1
C. Tran	1
Naval Environmental Prediction Research Facility, CA	1
Naval Intelligence Command, DC	1
Naval Oceanographic and Atmospheric Research Laboratory, CA	
M. Pastore	1
Naval Oceanographic and Atmospheric Research Laboratory, MS	
B. Adams	1
R. Fiddler	1
E. Franchi	1
R. Wagstaff	1
Naval Oceanographic Office, MS	1
Naval Personnel Research and Development Center, CA	1
Naval Postgraduate School, CA	
Superintendent	1
C. Therrien	1
Naval Research Laboratory, DC	
W. Gabriel	1
D. Steiger	1
E. Wald	1
N. Yen	1
Naval Surface Warfare Center, FL	
E. Linsenmeyer	1
D. Skinner	1

Addressee	No. of Copies
Naval Surface Warfare Center, MD	
P. Prendergast	1
Naval Surface Warfare Center, VA	
J. Gray	1
Naval Technical Intelligence Center, DC	
Commanding Officer	1
D. Rothenberger	1
Naval Undersea Warfare Center, FL	
Officer in Charge	1
Naval Weapons Center, CA	1
Office of the Chief of Naval Research, VA	
P. Abraham	1
N. Gerr	1
N. Harned	1
D. Johnson	1
R. Tipper	1
A. van Tilborg	1
Space and Naval Warfare Systems Command, DC	
R. Holland	1
U.S. Air Force, Maxwell Air Force Base, AL	
Air University Library	1
U.S. Department of Commerce, CO	
A. Spaulding	1
Vandenberg Air Force Base, CA	
R. Leonard	1

Addressee	No. of Copies
Brown University, RI	
Documents Library	1
Catholic University, DC	
J. McCoy	1
Drexel University, PA	
S. Kesler	1
Duke University, NC	
J. Krolik	1
Harvard University, MA	
Gordon McKay Library	1
Lawrence Livermore National Laboratory, CA	
L. Ng	1
Los Alamos National Laboratory, NM	1
Marine Biological Laboratory, MA	1
Massachusetts Institute of Technology, MA	
Barker Engineering Library	1
Northeastern University, MA	
C. Nikias	1
Pennsyvania State University, PA	
R. Hettche	1
E. Liszka	1
F. Symons	1
Princeton University, NJ	
S. Schwartz	1
Rutgers University, NJ	
S. Orfanidis	1
San Diego State University, CA	
F. Harris	1
Sandia National Laboratory, NM	
J. Claasen	1
Scripps Institution of Oceonography, CA	1
Syracuse University, NY	
D. Weiner	1
United Engineering Center, NY	
Engineering Societies Library	1
University of Colorado, CO	
L. Scharf	1
University of Connecticut, CT	
Wilbur Cross Library	1
C. Knapp	1
P. Willett	1
University of Florida, FL	
D. Childers	1
University of Hartford	
Science and Engineering Library	1
University of Illinois, IL	
D. Jones	1
A. Nehorai	1

Addressee	No. of Copies
University of Massachusetts, MA	
C. Chen	1
University of Michigan, MI	
Communications and Signal Processing Laboratory	1
W. Williams	1
University of Minnesota, MN	
M. Kaveh	1
University of Rhode Island, RI	
Library	1
G. Boudreaux-Bartels	1
S. Kay	1
D. Tufts	1
University of Rochester, NY	
E. Titlebaum	1
University of Southern California, CA	
W. Lindsey	1
A. Polydoros	1
University of Texas, TX	1
University of Washington, WA	
Applied Physics Laboratory	1
D. Lytle	1
J. Ritcey	1
R. Spindel	1
Villanova University, PA	
M. Amin	1
Woods Hole Oceanographic Institution, MA	
Director	1
T. Stanton	1
Yale University, CT	
Kline Science Library	1
P. Schultheiss	1

Addressee	No. of Copies
Analysis and Technology, CT	
Library	1
Analysis and Technology, VA	
D. Clark	1
Atlantic Aerospace Electronics Corp.	
R. Stahl	1
Bell Communications Research, NJ	
D. Sunday	1
Berkeley Research, CA	
S. McDonald	1
Bolt, Beranek, and Newman, CT	
P. Cable	1
Bolt, Beranek, and Newman, MA	
H. Gish	1
EDO Corporation, NY	
M. Blanchard	1
E G & G, VA	
D. Frohman	1
Engineering Technology Center	
D. Lerro	1
General Electric, MA	
R. Race	1
General Electric, NJ	
H. Urkowitz	1
Harris Scientific Services, NY	
B. Harris	1
Hughes Defense Communications, IN	
R. Kenefic	1
Hughes Aircraft, CA	
T. Posch	1
Kildare Corporation, CT	
R. Mellen	1
Lincom Corporation, MA	
T. Schonhoff	1
MITRE Corporation, CT	
S. Pawlukiewicz	1
Orincon Corporation, CA	
J. Young	1
Orincon Corporation, VA	
H. Cox	1
Prometheus, RI	
M. Barrett	1
J. Byrnes	1
Raytheon, RI	
R. Conner	1
S. Reese	1
Schlumberger-Doll Research, CT	
R. Shenoy	1

Addressee	No. of Copies
Science Applications International Corporation, CA	
C. Katz	1
Science Applications International Corporation, VA	
P. Mikhalevsky	1
Toyon Research, CA	
M. Van Blaricum	1
Tracor, TX	
T. Leih	1
B. Jones	1
K. Scarbrough	1
TRW, VA	
R. Prager	1
G. Maher	1
Westinghouse Electric, MA	
R. Kennedy	1
Westinghouse Electric, MD	
H. Newman	1
Westinghouse Electric, MD	
R. Park	1